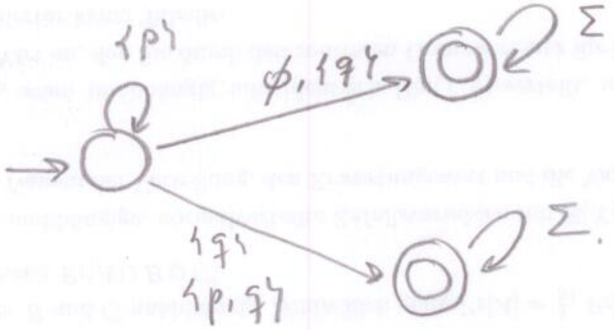
Exercise 1

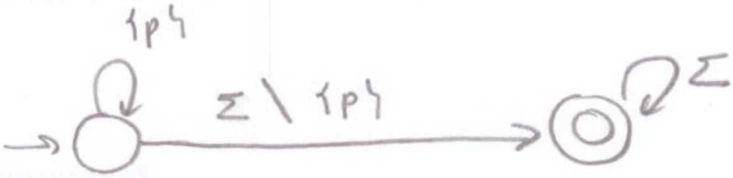
(a)

Formula	Satisfying	Not satisfying
$G(p U q)$	$\{q\}^\omega$	$\{p\}^\omega$
$GFp \rightarrow FGp$	$\{p\}^\omega$	$(\emptyset \{p\})^\omega$
$FG(Fp \vee FGq)$ $\equiv GF(p \vee Gq)$	$\{p\}^\omega$	$\emptyset^\omega$
$(pU\neg p) U (q \wedge \neg p)$	$\{q\}^\omega$	$\{p\}^\omega$

(b) $Gp \rightarrow pUq \equiv F\neg p \vee (pUq)$

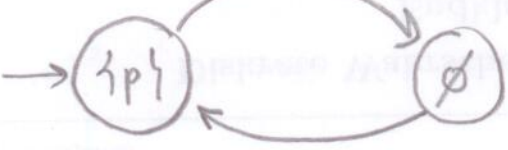


(or even



)

(c)



Exercise 2

The property is violated if there is a path of the following shape:

$$\begin{array}{ccccccccc}
s_0 & \rightsquigarrow & s_i & \rightsquigarrow & s_k & \rightsquigarrow & s_j & \rightsquigarrow & \cdots \\
 & & \models & & \models & & \models & & \\
 & & p & & p \vee q & & q & &
\end{array}$$

The existence of such a path is captured by the CTL formula

$$\phi = EF\,(p \wedge (p \vee q)\, EU\, q)$$

So the property ~~can be~~ is expressed by $\neg \phi$.

Exercise 3

(a) $\{ p, \neg p, Gp, \neg Gp, G\neg p, \neg G\neg p,$
$Gp \vee G\neg p, \neg(Gp \vee G\neg p) \}$

(b) With the help of $\neg G\phi \equiv F\neg\phi$ we can rewrite the subformulas as

$\{ p, \neg p, F\neg p, Gp, Fp, G\neg p, F\neg p \wedge Fp, \neg(F\neg p \wedge Fp) \}$

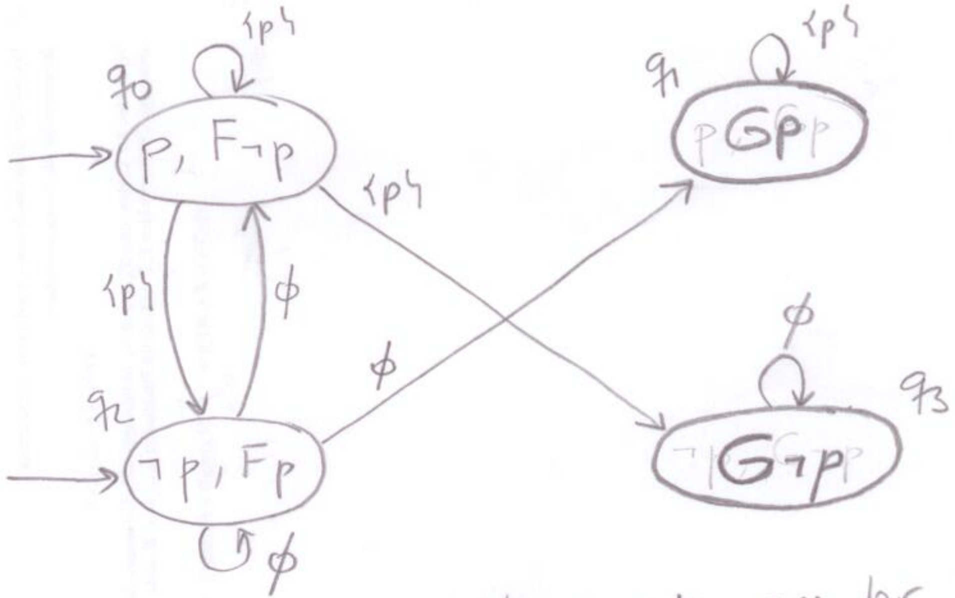
Since $p \wedge F\neg p \equiv \neg p \wedge Fp \equiv$ false, and $Gp \models p \models Fp$

states with a nonempty language are

$\{p, F\neg p, Fp \wedge F\neg p\}$ $\{p, Gp, \neg(Fp \wedge F\neg p)\}$

$\{\neg p, Fp, Fp \wedge F\neg p\}$ $\{\neg p, G\neg p, \neg(Fp \wedge F\neg p)\}$

Omitting formulas implied by others, we get

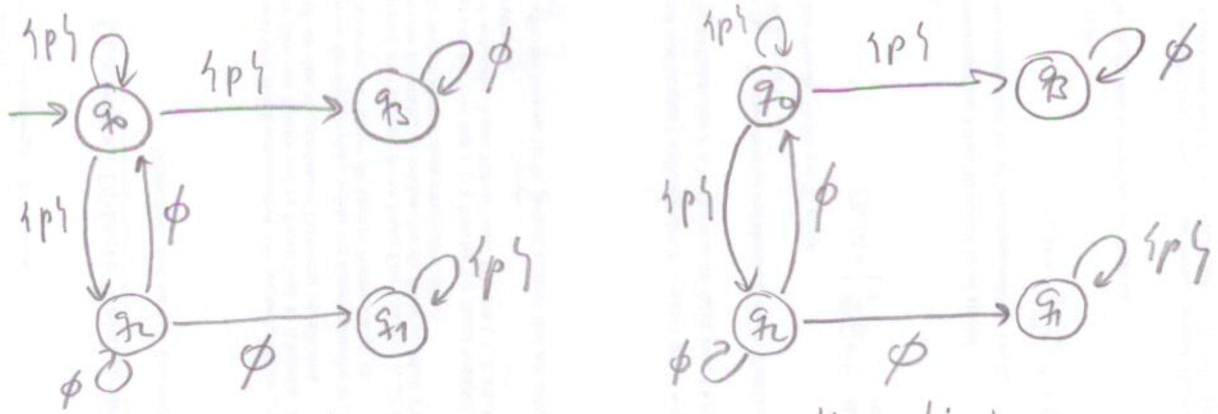


(c) We need two acceptance sets, one for Fp and a second for $F\neg p$.

The set for Fp contains the states satisfying p or $\neg Fp \equiv G\neg p$: $F_1 = \{q_0, q_1, q_3\}$

The set for $\neg Fp$ is, analogously: $F_2 = \{q_2, q_1, q_3\}$.

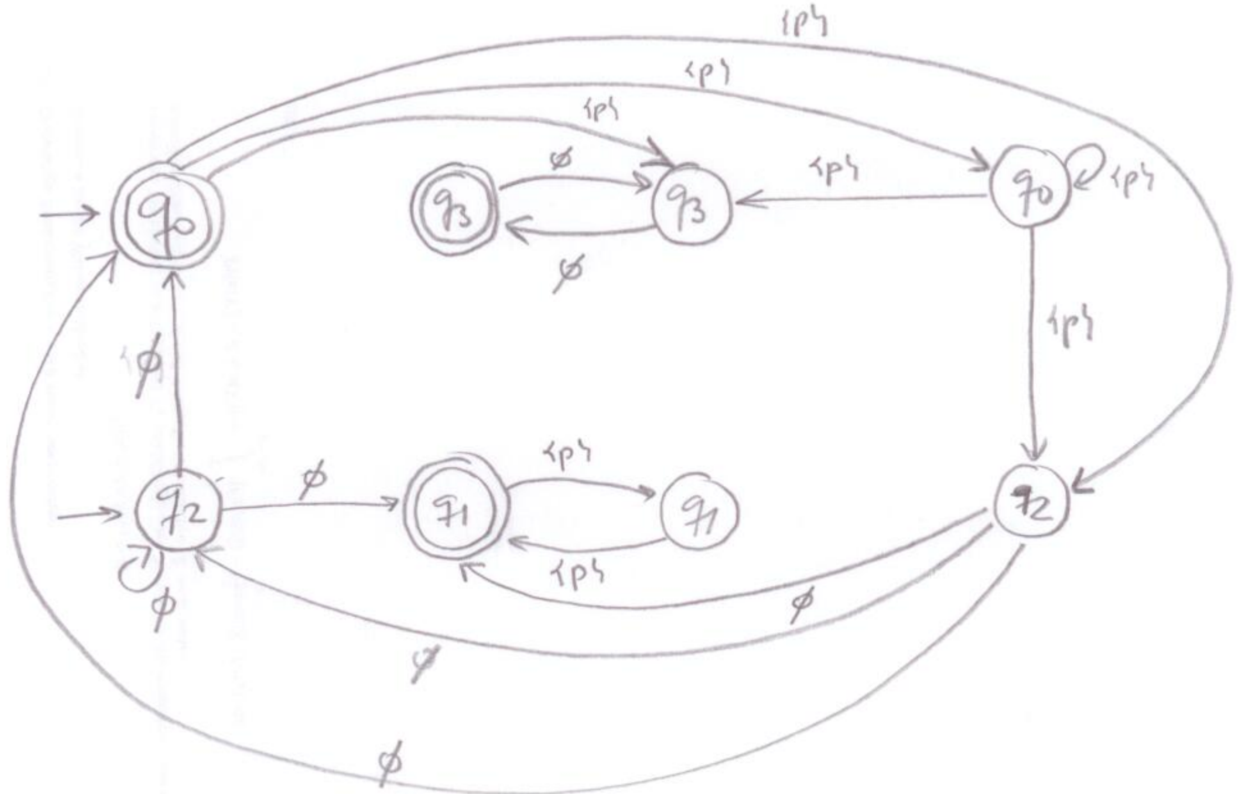
(d) The Büchi automaton contains two copies of the former automaton:



The transitions leaving q_0, q_1, q_3 in the first copy are redirected to the second copy

The transitions leaving q_1, q_2, q_3 in the second copy are redirected to the first copy

The final states are states q_0, q_1, q_3 in the first copy



Exercise 4

We rewrite the formula as

$EG\,\psi$ where $\psi = (\neg EX \neg p)\ EU\ q$

We compute bottom-up:

(i) $[\![q]\!] = \{q_2, q_5\}$

(ii) $[\![p]\!] = \{q_0, q_1, q_3, q_5\}$ $[\![\neg p]\!] = \{q_2, q_4, q_6\}$

(iii) $[\![EX \neg p]\!] = pre([\![\neg p]\!]) = \{q_1, q_3, q_0, q_2\}$

(iv) $[\![\neg EX \neg p]\!] = \{q_4, q_5, q_6\}$

(iv) $[\![\psi]\!] =$ smallest fixed point of

$$X = [\![q]\!] \cup (pre(X) \cap [\![\neg EX \neg p]\!])$$

$[\![\psi]\!] = \{q_2, q_5, q_4\}$

(v) $[\![EG\,\psi]\!] =$ largest fixed point of

$$X = [\![\psi]\!] \cap pre(X)$$

$[\![EG\,\psi]\!] = \{q_2, q_5, q_4\}$

Exercise 5

(a) The arrows indicate the reason for including a pair

$$\begin{tikzcd}
& (A,\alpha) \arrow[dl] \arrow[dr, shift left] & \\
(B,\beta) \arrow[d] & & (E,\zeta) \arrow[ul, shift left] \arrow[d] \\
(C,\gamma) \arrow[d] & & (D,\varepsilon) \arrow[d, shift right] \\
(D,\delta) \arrow[d, shift right] & & (C,\delta) \arrow[u, shift right] \\
(C,\varepsilon) \arrow[u, shift right] & &
\end{tikzcd}$$

(Other solutions are possible. For instance, E can also be simulated by β, instead of by ζ)

$$R = \{ (A,\alpha), (B,\beta), (C,\gamma), (C,\delta), (C,\varepsilon), (D,\delta), (D,\varepsilon), (E,\gamma) \}$$

(b) No. It suffices to show that K_3 has an infinite sequence that K_2 does not have.

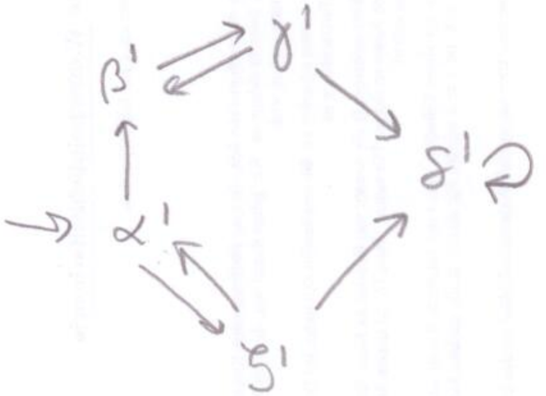
$$\begin{array}{cccccc} \emptyset & \{p\} & \emptyset & \emptyset & \{p\} & \cdots \\ 1 & 2 & 3 & 4 & 7 & \end{array}$$ is an infinite sequence of K_3.

K_2 does not have this sequence: after two consecutive "$\emptyset$" we are in δ or in ε, and we can't move to a state satisfying $\{p\}$.

(c) Yes, because K_2 can be "embedded" in K_3

$R = \{ (\alpha, 1), (\beta, 2), (\gamma, 3), (\delta, 4), (\varepsilon, 5), (\zeta, 6) \}$

(d)



The bisimulation is

$$R = \{ (\alpha, \alpha'), (\beta, \beta'), (\gamma, \gamma'), (\delta, \delta'), (\varepsilon, \delta'), (\zeta, \zeta') \}$$

The moves $\delta \to \varepsilon$ and $(\varepsilon \to \delta)$ are both matched by $\delta' \to \delta'$. The move $\delta' \to \delta'$ starting from (δ, δ') is matched by $\delta \to \varepsilon$, and starting from (ε, δ') by $\varepsilon \to \delta$.