Technische Universität München (I7)
Javier Esparza

Summer 2013

Model Checking – Endterm

Last name: ______________________________

First name: ______________________________

Student ID no.: ______________________________

Signature: ______________________________

- If you feel ill, let us know immediately.
- You have **90 minutes** to fill in all the required information and write down your solutions.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means**.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass (grade 4.0).
- Don't fill in the table below.
- Good luck!

Ex1	Ex2	Ex3	Ex4	Ex5	$\sum$

Exercise 1 **10P (4+4+2)**

In this exercise, let $AP = \{p, q\}$.

(a) For each of the following formulas, give two sequences of $(2^{AP})^\omega$, one that satisfies the formula, and one that does not.

- $\mathbf{G}(p \,\mathbf{U}\, q)$
- $(\mathbf{G}\,\mathbf{F}\, p) \rightarrow (\mathbf{F}\,\mathbf{G}\, p)$
- $\mathbf{F}\,\mathbf{G}((\mathbf{F}\, p) \vee \mathbf{F}\,\mathbf{G}\, q)$
- $(p \,\mathbf{U}\, \neg p)\,\mathbf{U}(q \wedge \neg p)$

(b) Give a Büchi automaton for the formula $(\mathbf{G}\, p) \rightarrow (p \,\mathbf{U}\, q)$. It is not necessary (and not advisable) to use the algorithm in the lecture. There exists a 3-state Büchi automaton for this formula (you may present a larger one).

(c) Consider the CTL formulas $\varphi_1 = \mathbf{AG}\,\mathbf{EF}\, p$ and $\varphi_2 = \mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\, p$. Give a Kripke structure K such that $K \models \varphi_1$ and $K \not\models \varphi_2$.

Exercise 2 **4P**

Consider the following property over the set of atomic propositions $AP = \{p, q\}$, expressed in English:

> For every path $s_0 s_1 s_2 \ldots$, if some state s_i $(i \geq 0)$ satisfies p and some state s_j such that $j \geq i$ satisfies q, then there is an index k satisfying $i \leq k \leq j$ such that s_k satisfies $\neg p \wedge \neg q$.

Express the property in CTL. Justify briefly your answer.

Exercise 3 **12P (1+5+3+3)**

Let $AP = \{p\}$ and the LTL formula over the set AP of atomic propositions: $\varphi = \neg((\mathbf{G}\, p) \vee (\mathbf{G}\, \neg p))$.

(a) List the subformulas of φ.

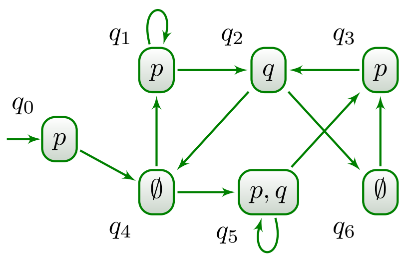
(b) Build the states and transitions of the generalized Büchi automaton obtained by the translation explained in the course. If you omit states without outgoing transitions, explain briefly why.

(c) Describe the acceptance condition of this generalized automaton.

(d) Transform the generalized Büchi automaton into an equivalent Büchi automaton.

Exercise 4 **6P**

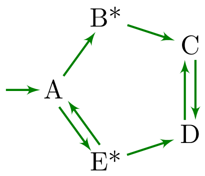
Consider the following Kripke structure over atomic propositions $AP = \{p, q\}$:



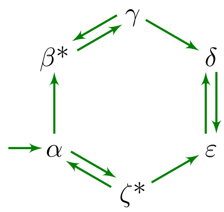
Compute the set of states satisfying $\mathbf{EG}((\mathbf{AX}\, p)\,\mathbf{EU}\, q)$ with the help of the bottom-up algorithm explained in the lecture. Give for each subformula the set of states satisfying it, starting at the leaves of the syntax tree. It is not necessary to describe how to compute fixpoints.

Exercise 5 **8P (2+2+2+2)**

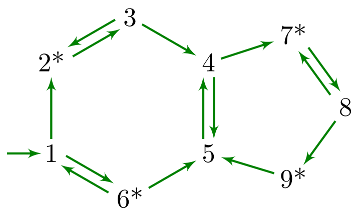
Consider the following three transition systems K_1, K_2 and K_3, over the set of atomic propositions $AP = \{p\}$. The proposition p holds for states B and E of K_1, states β and ζ of K_2 and states 2, 6, 7 and 9 of K_3. In the pictures, these states are marked with $*$.



K_1



K_2



K_3

(a) Give a simulation relation showing that K_2 simulates K_1.

(b) Does K_2 simulate K_3? If your answer is affirmative, give a simulation relation. If your answer is negative, explain why.

(c) Does K_3 simulate K_2? If your answer is affirmative, give a simulation relation. If your answer is negative, explain why.

(d) Give a Kripke structure that is bisimilar to K_2, and has fewer states. Give a bisimulation relation between the two structures.