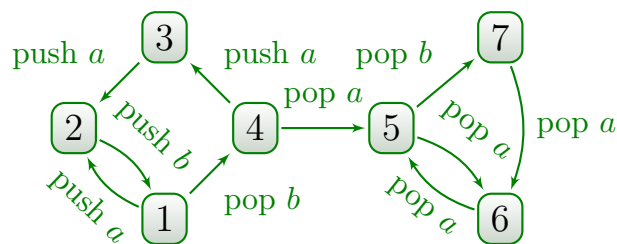


Model Checking – Exercise sheet 12

Exercise 12.1

Consider the pushdown system below, with stack alphabet $\Gamma = \{a, b\}$ where $\textcircled{1} \xrightarrow{\text{push } a} \textcircled{2}$, indicates the presence of transitions $1a \leftrightarrow 2aa$ and $1b \leftrightarrow 2ab$, and $\textcircled{4} \xrightarrow{\text{pop } a} \textcircled{5}$, indicates the presence of transition $4a \leftrightarrow 5$.



- Let $L = 7b^* = \{7, 7b, 7bb, 7bbb, \dots\}$. Construct the \mathcal{P} -automaton accepting $\text{pre}^*(L)$.
- Give the minimal automaton accepting the language of all stacks w such that $1w \in \text{pre}^*(L)$.

Exercise 12.2

Consider the following recursive program, where ? denotes a nondeterministic Boolean value:

```

    procedure main;
m0:   if ? then
        call a;
        else
        call b;
m1:   return;

    procedure a;
a0:   if ? then
        call b;
a1:   call b;
        else
        call a;
        end if;
a2:   return;
    
```

```

    procedure b;
b0:   if ? then
        call a;
b1:   if ? then
        call a;
        end if;
    end if;
b2:   return;

```

- (a) Model the program with a pushdown system.
- (b) Compute all configurations that can reach the program label `m1`.

Exercise 12.3

Consider the following recursive program with a global variable `g` and a local variable `l`:

```

    boolean g;

    procedure main(boolean l);
m0:   if l then
        call a;
    end if;
m1:   assert(g == l);
m2:   return;

    procedure a();
a0:   g := not g;
a1:   if not g then
        call a;
a2:   call a;
    end if;
a3:   return;

```

- (a) Model the program with a pushdown system, where the values of `g` and `l` are not initialized.
- (b) Compute all configurations that can reach the program label `m2`.
- (c) ★ Compute all configurations that are reachable from the program label `m0`.

Solution 12.1

(a) First note that the transitions of the pushdown system are as follows:

$$1a \rightarrow 2aa$$

$$1b \rightarrow 2ab$$

$$1b \rightarrow 4$$

$$2a \rightarrow 1ba$$

$$2b \rightarrow 1bb$$

$$3a \rightarrow 2aa$$

$$3b \rightarrow 2ab$$

$$4a \rightarrow 3aa$$

$$4b \rightarrow 3ab$$

$$4a \rightarrow 5$$

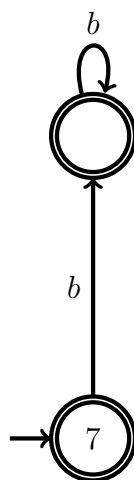
$$5b \rightarrow 7$$

$$5a \rightarrow 6$$

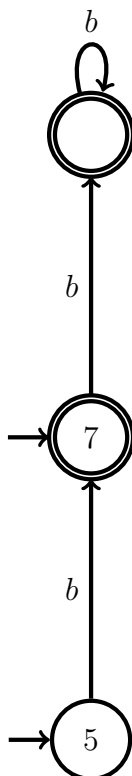
$$6a \rightarrow 5$$

$$7a \rightarrow 6.$$

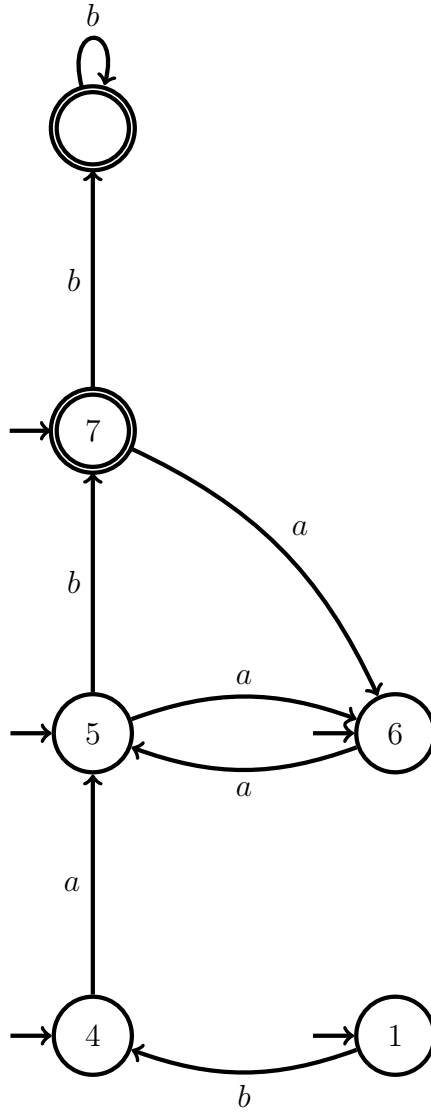
We are looking for $\text{pre}^*(L)$ where $L = 7b^*$. We construct the following \mathcal{P} -automaton for L :



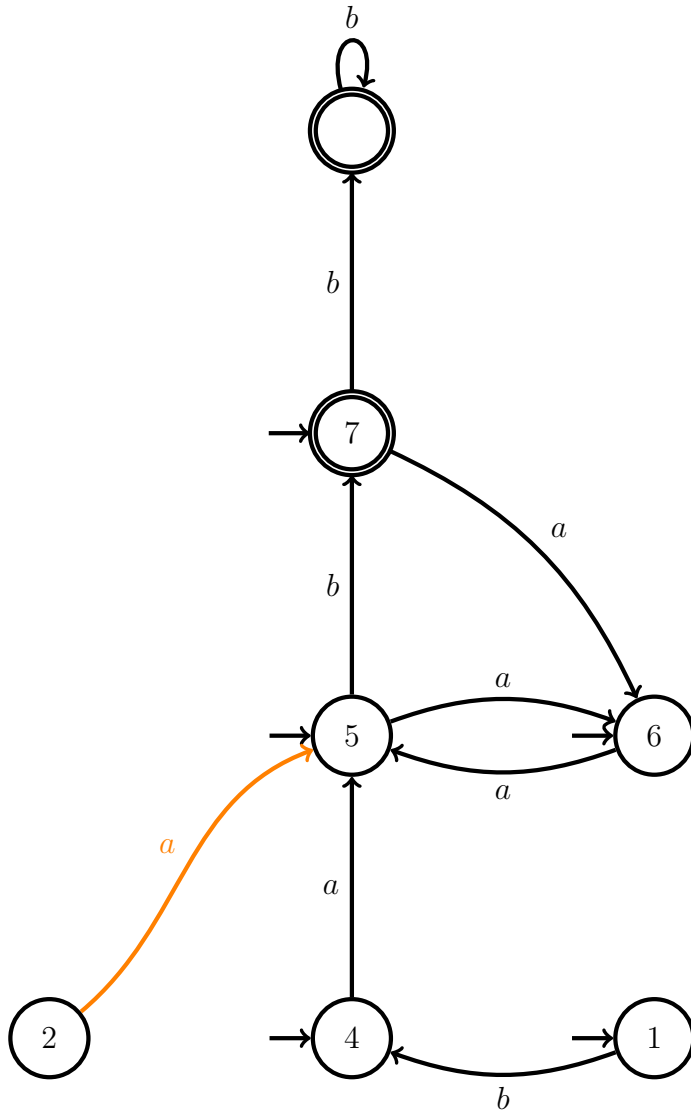
We apply the algorithm to compute $\text{pre}^*(L)$ on the above automaton \mathcal{A} . More precisely, if $q \xrightarrow{w} r$ in \mathcal{A} and if the pushdown system contains a rule $pA \rightarrow qw$, then we add a transition $p \xrightarrow{A} r$ to \mathcal{A} . For example, this is the case for $7 \xrightarrow{\varepsilon} 7$ and $5b \rightarrow 7$, so we add the transition $5 \xrightarrow{b} 7$:



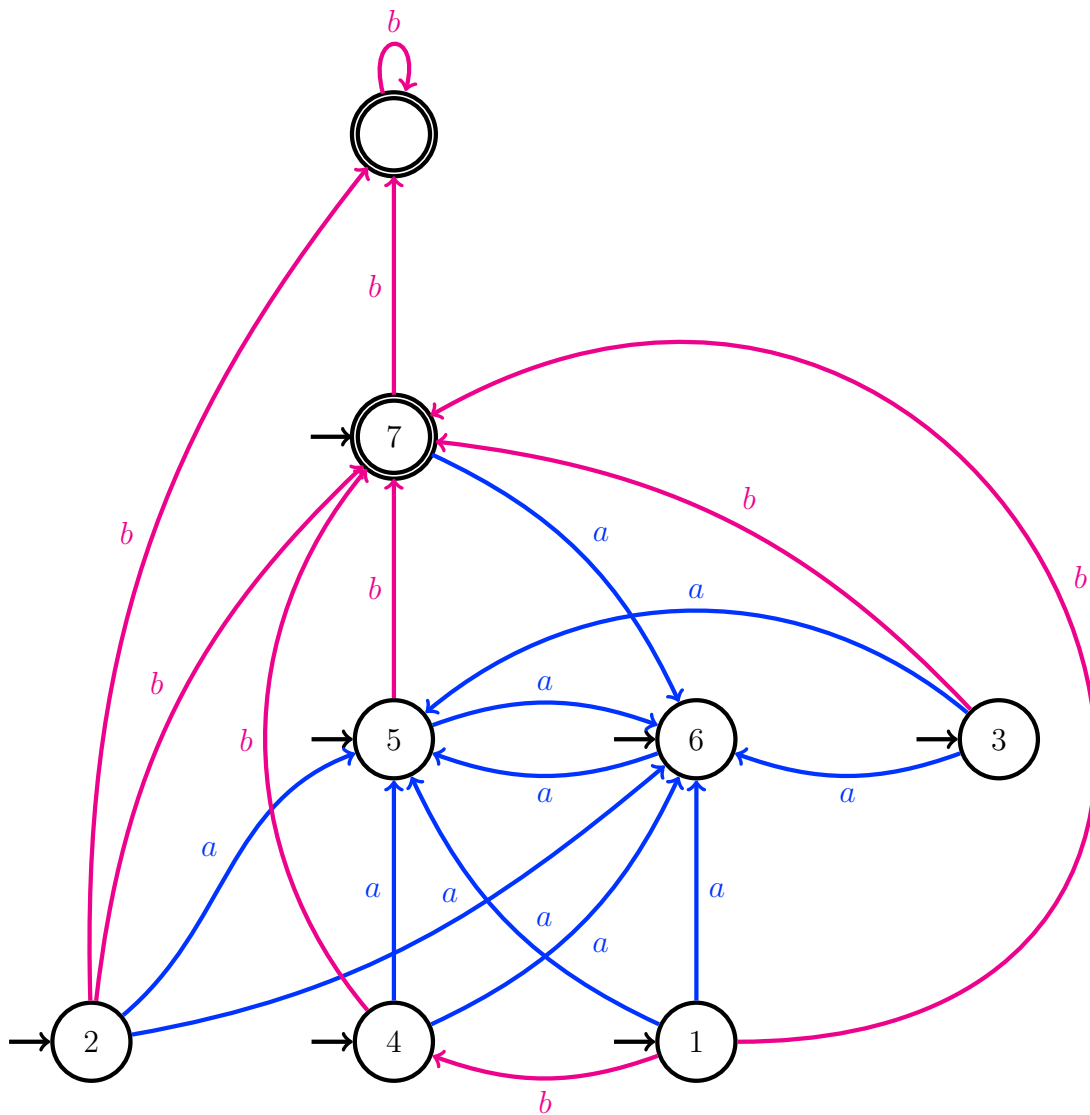
By repeatedly doing the same with rules $4a \rightarrow 5$, $6a \rightarrow 5$, $5a \rightarrow 6$, $7a \rightarrow 6$ and $1b \rightarrow 4$, we obtain:



From rule $2a \rightarrow 1ba$, we add the following (orange) transition:

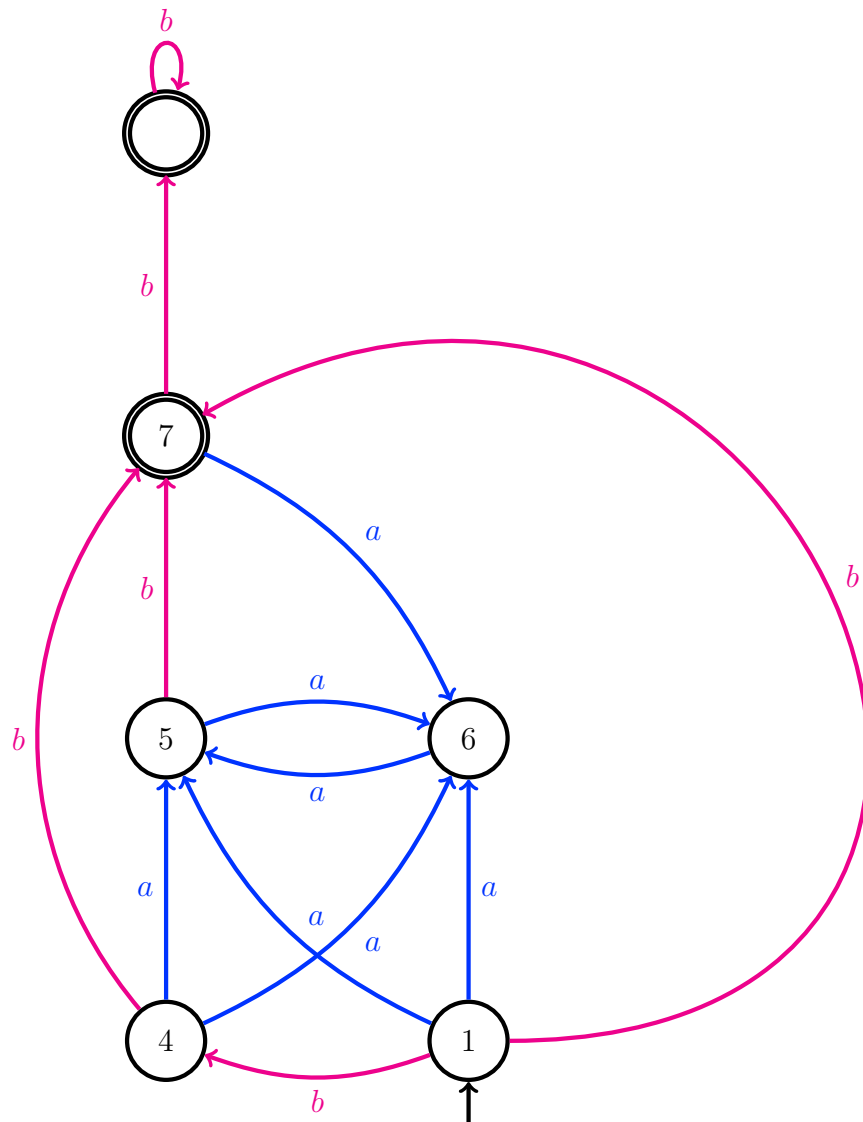


We repeat the process until we derive the following \mathcal{P} -automaton¹:

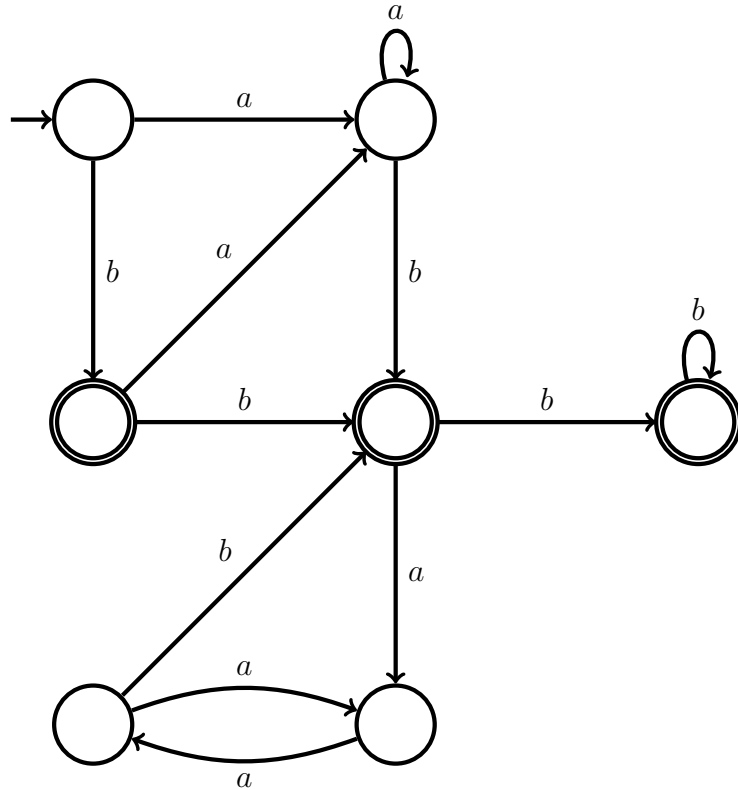


¹Blue and magenta are only used to help distinguishing a and b -transitions.

- (b) We are interested in the language accepted by the \mathcal{P} -automaton obtained in (a) starting from control-state 1. By removing the control-states which are non reachable, we obtain the following automaton:



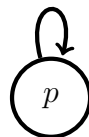
By determinizing and minimizing the above automaton, we derive:



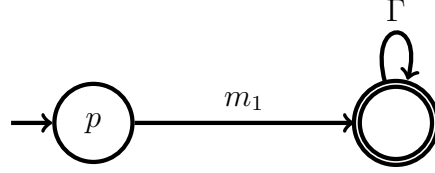
Solution 12.2

(a) Since the program has no global variable, the pushdown system has a single control-state, say p . The stack alphabet is $\Gamma = \{m_0, m_1, a_0, a_1, a_2, b_0, b_1, b_2\}$. The resulting pushdown system is:

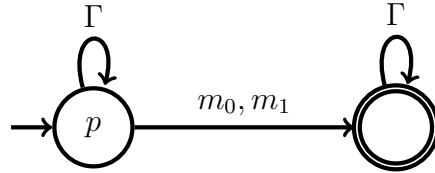
- $m_0 \rightarrow a_0m_1 \mid b_0m_1$
- $m_1 \rightarrow \varepsilon$
- $a_0 \rightarrow b_0a_1 \mid a_0a_2$
- $a_1 \rightarrow b_0a_2$
- $a_2 \rightarrow \varepsilon$
- $b_0 \rightarrow a_0b_1 \mid b_2$
- $b_1 \rightarrow a_0b_2 \mid b_2$
- $b_2 \rightarrow \varepsilon$



- (b) We are looking for $\text{pre}^*(L)$ where $L = p m_1 \Gamma^*$. We construct the following \mathcal{P} -automaton for L :



By applying the algorithm to compute $\text{pre}^*(L)$, we derive the following \mathcal{P} -automaton:

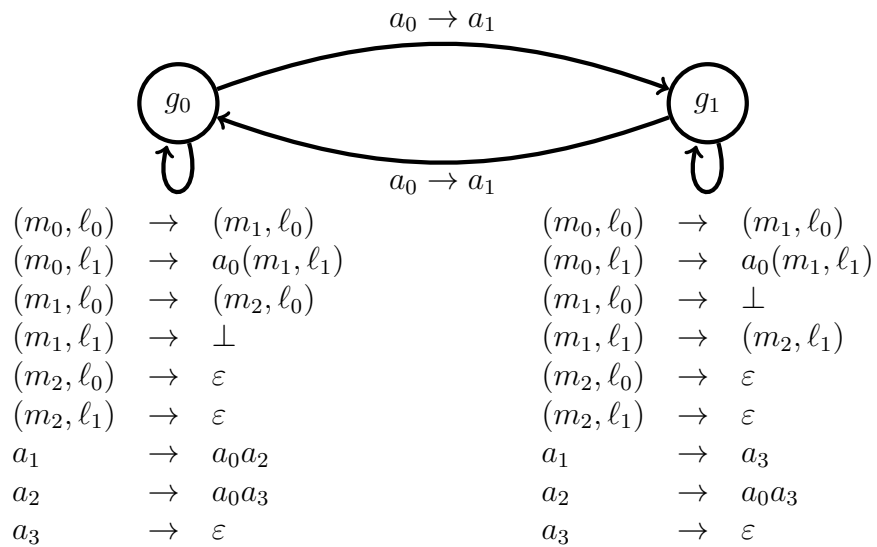


Solution 12.3

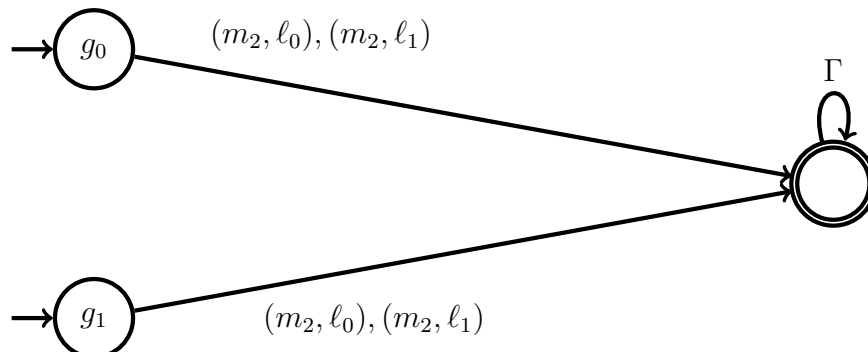
- (a) Since the program has a global boolean variable g , the pushdown system has two control-states g_0 and g_1 representing respectively $g = \text{false}$ and $g = \text{true}$. The stack alphabet is

$$\Gamma = \{(m_0, \ell_0), (m_0, \ell_1), (m_1, \ell_0), (m_1, \ell_1), (m_2, \ell_0), (m_2, \ell_1), a_0, a_1, a_2, a_3, \perp\}$$

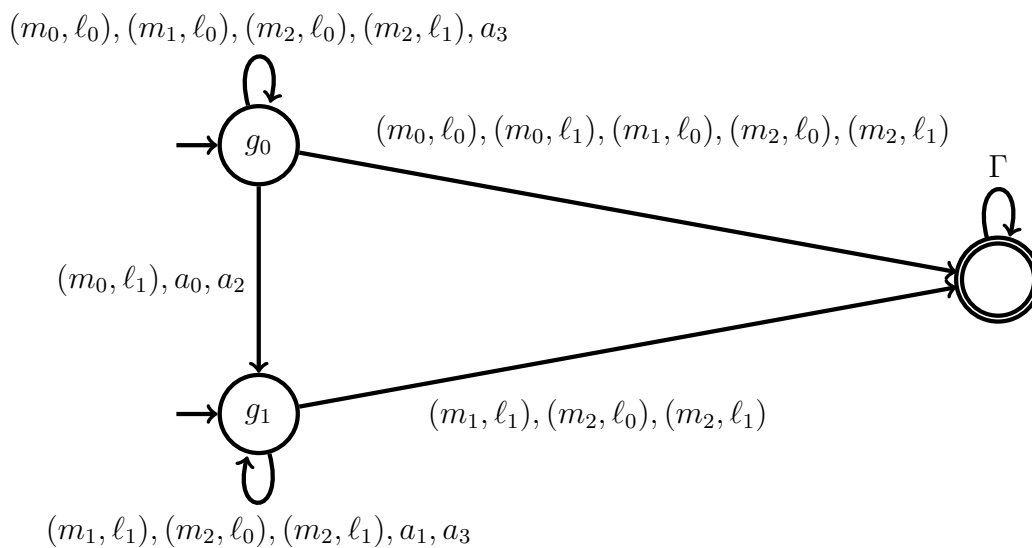
where \perp stands for an error, and (m_i, ℓ_j) stands for location m_i of main with $\ell = \text{true}$ if $j = 1$, and $\ell = \text{false}$ if $j = 0$. The resulting pushdown system is:



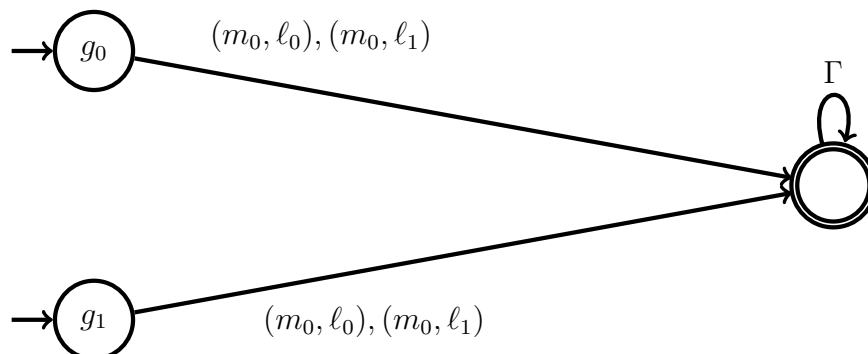
- (b) We are looking for $\text{pre}^*(L)$ where $L = (g_0 + g_1) ((m_2, \ell_0) + (m_2, \ell_1))\Gamma^*$. We construct the following \mathcal{P} -automaton for L :



By applying the algorithm to compute $\text{pre}^*(L)$, we derive the following \mathcal{P} -automaton:



- (c) We are looking for $\text{post}^*(L)$ where $L = (g_0 + g_1) ((m_0, \ell_0) + (m_0, \ell_1))\Gamma^*$. We construct the following \mathcal{P} -automaton for L :



By applying the algorithm to compute $\text{pre}^*(L)$, we derive the following \mathcal{P} -automaton:

