Technische Universität München (I7) **Summer Semester 2018**
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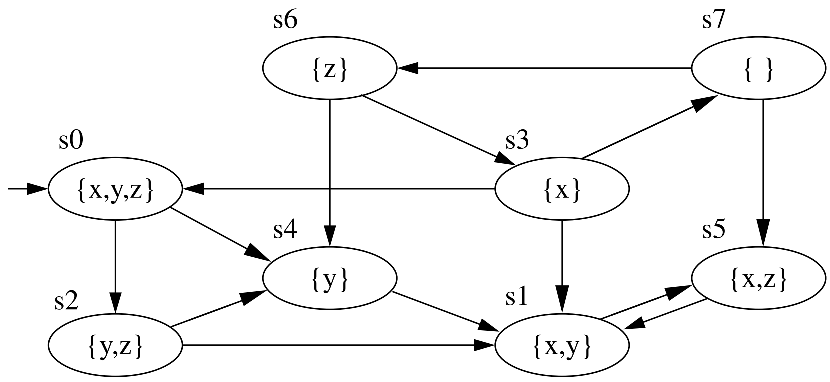
Model Checking – Exercise sheet 9

All exercises in this sheet are based on the following Kripke structure. The goal of this session is to understand the usage of Binary Decision Diagrams (BDDs) in CTL Model Checking. The CTL formula which we shall consider is $\phi = x \ \mathbf{EU} \ (y \wedge z)$. The following exercises will walk you through all the steps involved in the algorithm.



Exercise 10.1

1. Compute a BDD representing the states satisfying x. Encode each state $s_0, \ldots, s_7$ using three bits in the obvious way:

$$s_0 \mapsto 000, s_1 \mapsto 001, \ldots, s_7 \mapsto 111$$

   Draw first the full binary tree for the set, then merge all isomorphic subgraphs, and finally prune all redundant nodes, including the *false* node.

2. Compute a multi-BDD for the three sets of states satisfying y, z, and $y \wedge z$, respectively. For this, first construct the multi-BDD for the sets y and z, and then compute the BDD for $y \wedge z$ using the algorithm for intersection described in the lecture.

Exercise 10.2

Recall the steps involved in model checking of **EU**. $[\![p \ \mathbf{EU} \ q]\!]$ is the least fixed point of the sequence $\emptyset = X_0, \ X_1, \ X_2 \ \ldots$ where $X_{i+1} = \mu(q) \cup (\mu(p) \cap pre(X_i))$. Let TS be the transition relation of the Kripke structure.

$$TS = \{(s_0, s_2), (s_0, s_4), \ldots, (s_7, s_6)\}$$

1. Compute the set $X_1 = \mu(q) \cup (\mu(p) \cap pre(X_0))$ explicitly (not as a BDD).

2. Compute a multi-BDD for X_1 and $pre(X_1)$, proceeding as follows

   (a) Compute a multi-BDD for TS and $S \times X_1$. Think of states in X_1 being encoded by $a_0'a_1'a_2'$; states in S being encoded by $a_0a_1a_2$

   (b) Transform it into a multi-BDD for TS, $S \times X_1$, and $TS \cap (S \times X_1)$.

   (c) Use $\exists b \; B = B[b\backslash 0] \vee B[b\backslash 1]$ to compute $\exists a_0'a_1'a_2'$ TS $\cap$ $(S \times X_1)$

3. ★ Compute $X_2 = \mu(q) \cup (\mu(p) \cap pre(X_1))$, $X_3 = \mu(q) \cup (\mu(p) \cap pre(X_2))$, and compute a multi-BDD for X_1, X_2, X_3.