

Model Checking – Exercise sheet 8

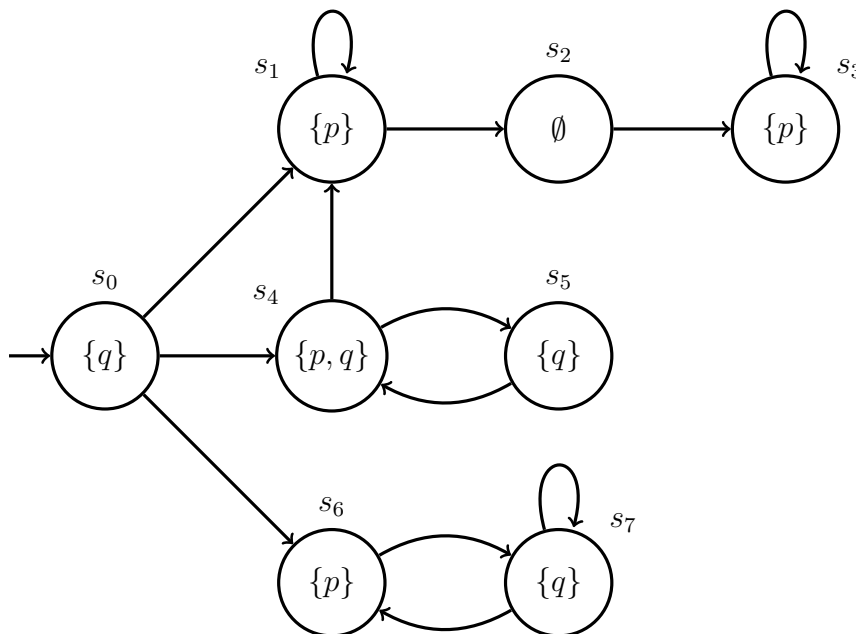
Exercise 8.1

Given two CTL formulas ϕ_1 and ϕ_2 , we write $\phi_1 \Rightarrow \phi_2$ iff for every Kripke structure \mathcal{K} we have $(\mathcal{K} \models \phi_1) \Rightarrow (\mathcal{K} \models \phi_2)$. Furthermore, we define an *implication graph* as a directed graph whose nodes are CTL formulas, and that contains an edge from ϕ_1 to ϕ_2 iff $\phi_1 \Rightarrow \phi_2$. Let $AP = \{p\}$.

- (a) Draw an implication graph with the nodes: **EFEF** p , **EGEG** p , **AFAF** p , **AGAG** p .
- (b) For each implication $\phi_1 \Rightarrow \phi_2$ obtained in (a), give a Kripke structure \mathcal{K} that satisfies ϕ_2 but not ϕ_1 , i.e. give a \mathcal{K} such that $\mathcal{K} \models \phi_2$ and $\mathcal{K} \not\models \phi_1$.
- (c) Add the following CTL formulas to the implication graph obtained in (a): **AFEF** p , **EFAF** p , **AGEG** p , **EGAG** p .
- (d) Complete the graph obtained in (c) with the nodes: **AGAF** p , **AFAG** p , **AGEF** p , **EGAF** p , **AFEG** p , **EFAG** p , **EFEG** p , **EGEF** p .

Exercise 8.2

Consider the following Kripke structure over $AP = \{p, q\}$:



- (a) Compute $\llbracket \mathbf{EG}q \rrbracket$ and $\llbracket \mathbf{EF}q \rrbracket$.
- (b) Compute $\llbracket \mathbf{AGAF}p \rrbracket$ and $\llbracket \mathbf{EFAG}\neg q \rrbracket$.