Model Checking – Exercise sheet 4

Exercise 4.1
Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give Büchi automata recognizing the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

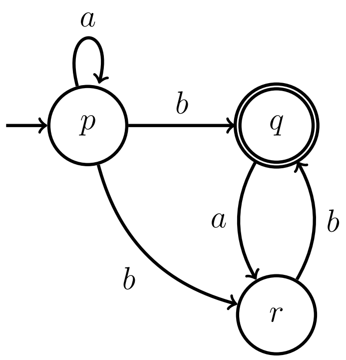
(b) $(\mathbf{GF}p) \rightarrow (\mathbf{F}q)$

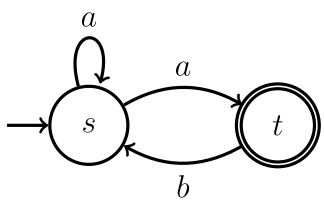
(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p\ \mathbf{U}\ (p \rightarrow q))$

(e) $\mathbf{F}q \rightarrow (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

Exercise 4.2
Let A and B be the following Büchi automata over $\Sigma = \{a, b\}$. Construct a Büchi automaton C such that $\mathcal{L}(C) = \mathcal{L}(A) \cap \mathcal{L}(B)$. Moreover, say whether there exists a deterministic Büchi automaton recognizing $\mathcal{L}(C)$. Justify your answer.





Exercise 4.3
Let $AP = \{p, q, r\}$ and $\Sigma = 2^{AP}$. For every $\sigma \in \Sigma^\omega$, let

$$P_\sigma = \{i \in \mathbb{N} : p \in \sigma(i)\},$$
$$Q_\sigma = \{i \in \mathbb{N} : q \in \sigma(i)\}.$$

We say that a sequence $\sigma \in \Sigma^\omega$ is *good* if there exists an injective function $f \colon P_\sigma \to Q_\sigma$ such that $i \leq f(i)$ for every $i \in P_\sigma$. Let $L = \{\sigma \in \Sigma^\omega : \sigma \text{ is good}\}$. Intuitively, L is the language of sequences where each occurrence of p is matched by a later occurrence of q.

(a) Show that $L \cap [\![\mathbf{GF}p]\!] = [\![(\mathbf{GF}p) \land (\mathbf{GF}q)]\!]$.

(b) Show that $L \cap \{p\}^*\{q\}^*\emptyset^\omega = L'$ where $L' = \{\{p\}^m\{q\}^n\emptyset^\omega : m \leq n\}$.

(c) Show that there is no Büchi automata recognizing L'. [Hint:]

(d) Show that there is no Büchi automata recognizing L. [Hint:]