Model Checking – Exercise sheet 2

Exercise 2.1

Let $\varphi = \mathbf{GF}p \to \mathbf{FG}(q \vee r)$ and $\psi = (r \; \mathbf{U} \; \mathbf{X}p) \; \mathbf{U} \; (q \wedge \neg\mathbf{XX}s)$ be LTL formulas over the atomic propositions $AP = \{p, q, r, s\}$. Say whether the following sequences satisfy φ and ψ. Justify your answers.

(a) $\emptyset^{\omega}$

(b) $\{p, q, r, s\}^{\omega}$

(c) $\{p\}^{\omega}$

(d) $\{q\}^{\omega}$

(e) $\{p, q\}^{\omega}$

(f) $\{r\}\emptyset\{p, q, s\}^{\omega}$

(g) $\{r\}\emptyset(\{p, q\}\{r, s\})^{\omega}$

(h) $\{r\}\emptyset\{p\}\{q, r\}(\{p, s\}\emptyset)^{\omega}$

(i) $\{r\}\emptyset\{p\}\{p, q, r\}(\{s\}\emptyset)^{\omega}$

(j) $\{q, r\}\emptyset\{p, q\}\emptyset\{r, s\}^{\omega}$

Exercise 2.2

Let $AP = \{s, r, g\}$ be actions of a process: **s**ending a message, **r**eceiving a message, and **g**iving a result, respectively. Specify the following properties in LTL, and give example sequences that satisfy and violate the formulas.

(a) The process always gives a result.

(b) The process stops communicating after giving its result.

(c) The process sends infinitely many messages.

(d) The process only gives a result once.

(e) The process receives a message after it sends one.

(f) The process does nothing until it receives a message.

Exercise 2.3

Let $AP = \{p, q\}$. An LTL formula is a tautology if it is satisfied by all sequences over 2^{AP}. Which of the following LTL formulas are tautologies? Justify each answer with a counterexample or a proof.

(a) $\mathbf{G}p \to \mathbf{F}p$

(b) $\mathbf{G}(p \to q) \to (\mathbf{G}p \to \mathbf{G}q)$

(c) $\mathbf{FG}p \vee \mathbf{FG}\neg p$

(d) $\neg\mathbf{F}p \to \mathbf{F}\neg\mathbf{F}p$

(e) $\neg(p \mathbf{\ U\ } q) \leftrightarrow (\neg p \mathbf{\ U\ } \neg q)$

(f) $(\mathbf{G}p \to \mathbf{F}q) \leftrightarrow (p \mathbf{\ U\ } (p \vee q))$