Technische Universität München (I7) **Summer Semester 2016**
J. Křetínský / D. Suwimonteerabuth **19.05.2016**

Model Checking – Sample Solution 5

Exercise 5.1

Extend the definition of NNF to include $\mathbf{F}$ and $\mathbf{G}$, extend the corresponding $Sub(\phi)$:

- if $\mathbf{F}\,\phi_1 \in Sub(\phi)$ then $\phi_1 \in Sub(\phi)$
- if $\mathbf{G}\,\phi_1 \in Sub(\phi)$ then $\phi_1 \in Sub(\phi)$,

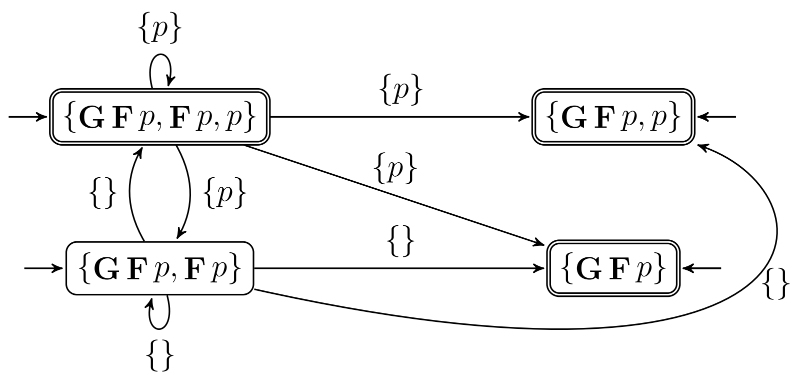
and extend rules for transitions as follows: $(M, \sigma, M') \in \Delta$ iff $\sigma = M \cap AP$ and

- if $\mathbf{F}\,\phi_1 \in Sub(\phi)$, then $\mathbf{F}\,\phi_1 \in M$ iff $\phi_1 \in M$ or $\mathbf{F}\,\phi_1 \in M'$
- if $\mathbf{G}\,\phi_1 \in Sub(\phi)$, then $\mathbf{G}\,\phi_1 \in M$ iff $\phi_1 \in M$ and $\mathbf{G}\,\phi_1 \in M'$

Also, the acceptance condition must be extended for $\mathbf{F}$: $\mathcal{F}$ contains a set F_ψ, for every subformula ψ of the form $\mathbf{F}\,\phi_1$, where

$$F_\psi = \{M \in CS(\phi) \mid \phi_1 \in M \text{ or } \neg(\mathbf{F}\,\phi_1) \in M\}$$

The translated Büchi automaton for $\phi = \mathbf{G}\,\mathbf{F}\,p$ is below. Notice that the initial states must contain $\mathbf{G}\,\mathbf{F}\,p$, and from the translation rule successors of states with $\mathbf{G}\,\mathbf{F}\,p$ must also contain $\mathbf{G}\,\mathbf{F}\,p$. So, it is not necessary to construct states without $\mathbf{G}\,\mathbf{F}\,p$.



Exercise 5.2

Translate ϕ into an NNF formula:

$$\begin{aligned}\phi &= \mathbf{G}\,((\mathbf{X}(p\,\mathbf{U}\,q)) \rightarrow ((\neg p \wedge \mathbf{F}\,q) \vee (q\,\mathbf{U}\,\mathbf{X}\,q))) \\ &\equiv \mathbf{G}\,((\mathbf{X}(\neg p\,\mathbf{R}\,\neg q)) \vee ((\neg p \wedge \mathbf{F}\,q) \vee (q\,\mathbf{U}\,\mathbf{X}\,q)))\end{aligned}$$

(a) Let $\phi_1 = \neg p \ \mathbf{R} \ \neg q$, $\phi_2 = \neg p \wedge \mathbf{F}\, q$, and $\phi_3 = q \ \mathbf{U} \ \mathbf{X}\, q$. We have $\phi = \mathbf{G}(\mathbf{X}\, \phi_1 \vee (\phi_2 \vee \phi_3))$ and $Sub(\phi) = \{\mathbf{true}, \phi, \mathbf{X}\, \phi_1 \vee (\phi_2 \vee \phi_3), \mathbf{X}\, \phi_1, \phi_2 \vee \phi_3, \phi_1, \phi_2, \phi_3, \mathbf{F}\, q, \mathbf{X}\, q, p, q\} \cup \{\mathbf{false}, \neg\phi, \neg(\mathbf{X}\, \phi_1 \vee (\phi_2 \vee \phi_3)), \neg\, \mathbf{X}\, \phi_1, \neg(\phi_2 \vee \phi_3), \neg\phi_1, \neg\phi_2, \neg\phi_3, \neg\, \mathbf{F}\, q, \neg\, \mathbf{X}\, q, \neg p, \neg q\}$.

(b) Only $\phi, \mathbf{X}\, \phi_1, \phi_1, \phi_3, \mathbf{F}\, q, \mathbf{X}\, q, p, q$ can independently form consistent states. So, $|CS(\phi)| = 2^8 = 256$ states

(c) $\mathcal{F} = \{F_{q \mathbf{U} \mathbf{X}\, q}, F_{\mathbf{F}\, q}\}$

(d) $\{\phi\} \in F_{q \mathbf{U} \mathbf{X}\, q}$ and $\{\phi\} \in F_{\mathbf{F}\, q}$

(e) $\{\phi\}$ is reachable because it is an initial state, and it has no successors because $\mathbf{X}\, \phi_1 \vee (\phi_2 \vee \phi_3) \notin \{\phi\}$.

(f) $\{\phi, q \ \mathbf{U} \ \mathbf{X}\, q\}$

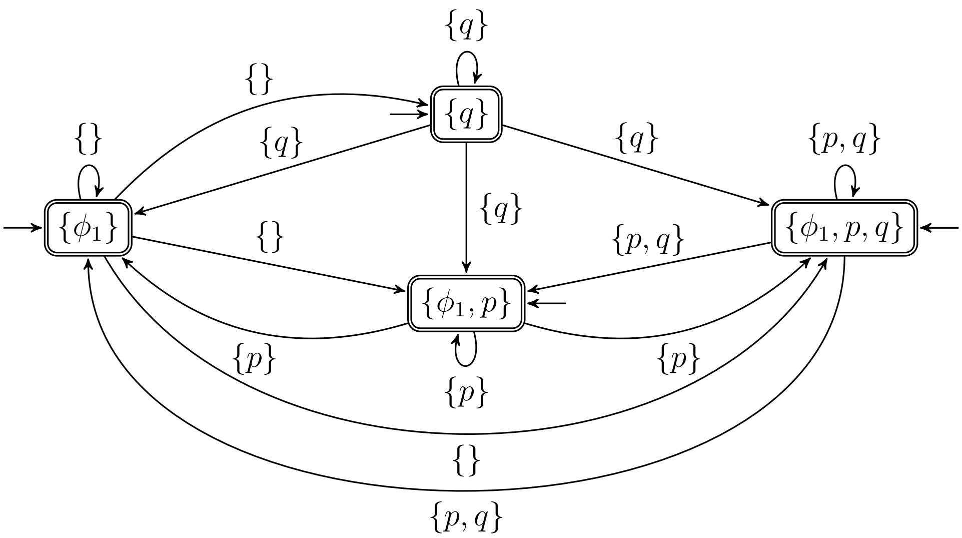
(g) $\{\phi, q, q \ \mathbf{U} \ \mathbf{X}\, q, \mathbf{F}\, q, \mathbf{X}\, q\}$

Exercise 5.3

(a) $\phi = \mathbf{G}\, \mathbf{F}(p \wedge (p \ \mathbf{U} \ (\neg p \wedge q)))$

(b) Construct a Büchi automaton for $\neg\phi$ by using e.g. the translation in the lecture.

(c) Let $\phi_1 = \neg p \ \mathbf{R} \ (p \vee \neg q)$. Note that ϕ_1, p, q are enough to form consistent sets, i.e. we assume that ϕ and $\neg p \vee (\neg p \ \mathbf{R} \ (p \vee \neg q))$ are implicitly in every state. So, $CS(\phi) = 2^{\{\phi_1, p, q\}}$. However, we know that $\{p\}$ and $\{p, q\}$ have no successors because of $\mathbf{G}$, and $\{\}$ and $\{\phi_1, q\}$ have no successors because of $\mathbf{R}$.



(d) Notice that $\neg\phi \equiv \mathbf{F}\, \mathbf{G}(\neg p \vee (\neg p \ \mathbf{R} \ (p \vee \neg q)))$. It suffices to add a self-looping initial state and transitions from it to all states in (c).