Technische Universität München (I7) **Summer Semester 2017**
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Model Checking – Exercise sheet 4

Exercise 4.1

Which of the following pairs ϕ and ψ of LTL formulas are equivalent, i.e. for which pairs is it true that for all sequences σ, the equivalence $\sigma \models \phi \Leftrightarrow \sigma \models \psi$ holds? In each case, give a proof if the equivalence holds or a counterexample otherwise.

(a) $\phi = \mathbf{F}\, p \wedge \mathbf{G}\, q$ $\qquad \psi = \mathbf{F}(p \wedge \mathbf{G}\, q)$
(b) $\phi = \mathbf{F}\, p \wedge \mathbf{G}\, q$ $\qquad \psi = \mathbf{G}(\mathbf{F}\, p \wedge q)$
(c) $\phi = (p \;\mathbf{U}\; q) \;\mathbf{U}\; q$ $\qquad \psi = p \;\mathbf{U}\; q$
(d) $\phi = (p \;\mathbf{U}\; q) \;\mathbf{U}\; (q \;\mathbf{U}\; r)$ $\qquad \psi = p \;\mathbf{U}\; r$

Exercise 4.2

Let AP be a set of atomic propositions and **NNF** be the set of NNF formulae over AP. Recall from the lecture that a formula is in **NNF** if negations only occur in front of atomic propositions, and only the following operators are allowed in the formula: $\vee, \wedge, \mathbf{X}, \mathbf{U}, \mathbf{R}$.

(a) Let $\mathbf{NNF}_{-\mathbf{R}}$ be the set of NNF formulae in which the operator $\mathbf{R}$ does not occur. Show that for any formula $\phi \in \mathbf{NNF}_{-\mathbf{R}}$ and $\sigma \in (2^{AP})^\omega$ such that $\sigma \models \phi$, there exists an integer $n_\phi(\sigma)$ such that $\sigma(0) \dots \sigma(n_\phi(\sigma))$ characterizes whether $\sigma \models \phi$ or not. Formally, prove that

$$\sigma \models \phi \Leftrightarrow \sigma(0) \dots \sigma(n_\phi(\sigma))\sigma' \models \phi$$

for any $\sigma' \in (2^{AP})^\omega$.

(b) Let $\mathbf{NNF}_{-\mathbf{X}}$ be the set of NNF formulae in which the operator $\mathbf{X}$ does not occur. Show that any formula $\phi \in \mathbf{NNF}_{-\mathbf{X}}$ cannot distinguish $\sigma \in (2^{AP})^\omega$ and $D(\sigma) = \sigma(0)\sigma(0)\sigma(1)\sigma(1)\dots$, i.e.

$$\sigma \models \phi \Leftrightarrow D(\sigma) \models \phi$$

Exercise 4.3

For each set of atomic propositions AP and LTL formula over AP below, construct a Büchi automaton $\mathcal{B}$ with the alphabet 2^{AP} such that $\mathcal{L}(\mathcal{B}) = [\![\phi]\!]$.

(a) $\mathbf{G}(p \wedge \mathbf{F}\, q)$, where $AP = \{p, q\}$

(b) $\mathbf{G}(p \rightarrow \mathbf{X}(q \;\mathbf{U}\; r))$, where $AP = \{p, q, r\}$

(c) $p \;\mathbf{U}\; (q \;\mathbf{U}\; r))$, where $AP = \{p, q, r\}$