Technische Universität München (I7) **Summer Semester 2017**
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Model Checking – Exercise sheet 2

Exercise 2.1

Let $AP = \{p, q, r, s\}$. Find out whether the following sequences satisfy the formulae $\phi = \mathbf{G}\,\mathbf{F}\,p$ and $\psi = \mathbf{F}\,\mathbf{G}(p \land \neg q \to (r\ \mathbf{U}\ s))$:

- $\emptyset^\omega$
- $\{p\}^\omega$
- $\{p, q\}^\omega$
- $\{p\}\{q\}^\omega$
- $(\{p\}\{q\})^\omega$
- $(\{p\}\{p, q\})^\omega$
- $(\{p\}\{r\})^\omega$
- $(\{q\}\{s\})^\omega$
- $(\{p, s\}\{q\})^\omega$
- $(\{p\}\{q\}\{r\}\{s\})^\omega$
- $(\{p, q\}\{q\}\{r\}\{s\})^\omega$
- $(\{p, r\}\{q\}\{r\}\{s\})^\omega$
- $(\{p, r\}\{q, r\}\{r\}\{r, s\})^\omega$

Exercise 2.2

Let $AP = \{s, r, g\}$ be actions of a process: **s**ending a message, **r**eceiving a message, and **g**iving a result, respectively. Specify the following properties in LTL, and give example sequences that satisfy and violate the formulae.

1. The process always gives a result.
2. The process stops communicating after giving its result.
3. The process sends infinitely many messages.

4. The process only gives a result only once.

5. The process receives a message after it sends one.

6. The process does nothing until it receives a message.

Exercise 2.3

Let ϕ and ψ be LTL formulae over a set of atomic propositions AP and $\sigma \in (2^{AP})^\omega$ a valuation sequence. Prove the following equivalences:

$$
\begin{array}{lcl}
\sigma \models \mathbf{G}\,\phi & \Leftrightarrow & \forall i : \sigma^i \models \phi \\
\sigma \models \phi\,\mathbf{R}\,\psi & \Leftrightarrow & \forall i : \forall j < i : w^j \not\models \phi \rightarrow \sigma^i \models \psi \\
\sigma \models \neg(\phi\,\mathbf{U}\,\psi) & \Leftrightarrow & \sigma \models \neg\phi\,\mathbf{R}\,\neg\psi \\
\sigma \models \phi\,\mathbf{U}\,\psi & \Leftrightarrow & \sigma \models \psi \vee (\phi \wedge \mathbf{X}(\phi\,\mathbf{U}\,\psi)) \\
\sigma \models \phi\,\mathbf{R}\,\psi & \Leftrightarrow & \sigma \models \mathbf{G}\,\psi \vee (\psi\,\mathbf{U}\,(\phi \wedge \psi))
\end{array}
$$

Exercise 2.4

Let $\mathcal{K}_1 = (S, \rightarrow_1, r, AP, \nu)$ and $\mathcal{K}_2 = (S, \rightarrow_2, r, AP, \nu)$ be two almost identical Kripke structures, except for the transition relations. We write $\mathcal{K}_1 \leq \mathcal{K}_2$ if the transition relation $\rightarrow_2$ contains $\rightarrow_1$, i.e. $\rightarrow_1 \subseteq \rightarrow_2$.

Show that if $\mathcal{K}_1 \leq \mathcal{K}_2$, then for any LTL formula ϕ:

$$\mathcal{K}_2 \models \phi \Rightarrow \mathcal{K}_1 \models \phi\ .$$