Technische Universität München (I7) **Summer Semester 2016**
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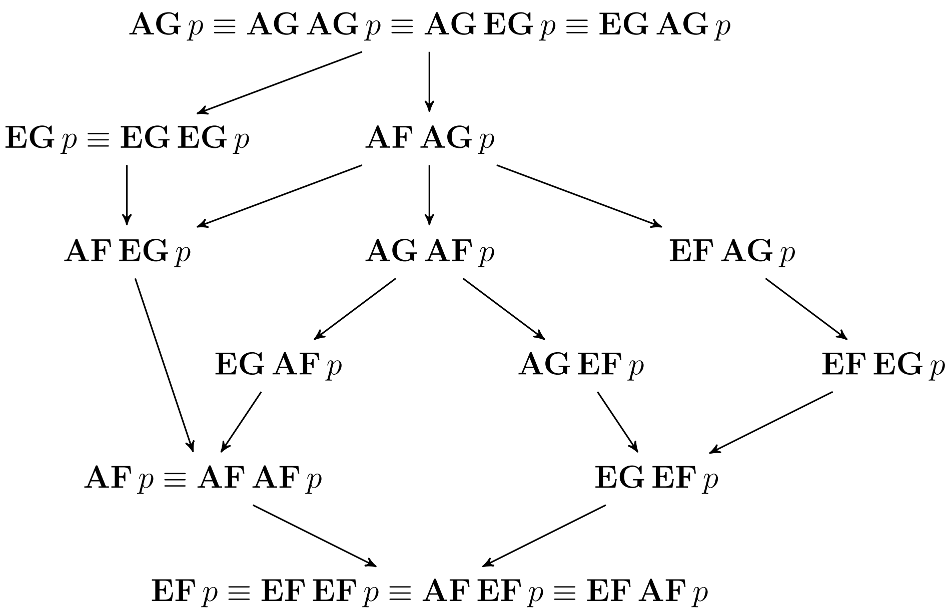
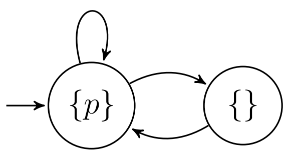
Model Checking – Sample Solution 8

Exercise 8.1

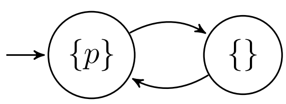
The implication graph for (a), (b), and (d) are as follows. Notice that the relation is transitive and all transitive edges are omitted.



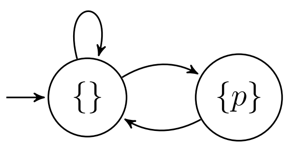
(c) The following Kripke structure satisfies $\mathbf{EG}\,p$, but not $\mathbf{AG}\,p$.



The following Kripke structure satisfies $\mathbf{AF}\,p$, but not $\mathbf{EG}\,p$.



The following Kripke structure satisfies $\mathbf{EF}\,p$, but not $\mathbf{AF}\,p$.



Exercise 8.2

a) For $[\![\mathbf{EG}\, q]\!]$, compute the largest fixed point from the sequence $\pi^0(S), \pi^1(S), \pi^2(S), \ldots$ where $\pi^0(S) = S$ and $\pi^{i+1}(S) = \mu(q) \cap pre(\pi^i(S))$. We have $\pi^0(S) = S$, $\pi^1(S) = \{s_0, s_2, s_5, s_7\}$, $\pi^2(S) = \{s_7\}$, and $\pi^3(S) = \pi^2$. Therefore, $[\![\mathbf{EG}\, q]\!] = \{s_7\}$.

For $[\![\mathbf{EF}\, q]\!]$, compute the smallest fixed point from the sequence $\xi^0(\emptyset), \xi^1(\emptyset), \xi^2(\emptyset), \ldots$ where $\xi^0(\emptyset) = \emptyset$ and $\xi^{i+1}(\emptyset) = \mu(q) \cup pre(\xi^i(\emptyset))$. We have $\xi^0(\emptyset) = \emptyset$, $\xi^1(\emptyset) = \{s_0, s_2, s_5, s_7\}$, $\xi^2(\emptyset) = \{s_0, s_1, s_2, s_4, s_5, s_6, s_7\}$, and $\xi^3(\emptyset) = \xi^2(\emptyset)$. Therefore, $[\![\mathbf{EF}\, q]\!] = S \setminus \{s_3\}$.

b) Notice that $\mathbf{AG}\,\mathbf{AF}\, p \equiv \neg\,\mathbf{EF}\,\mathbf{EG}\,\neg p$ and $\mathbf{AF}\,\mathbf{AG}\, p \equiv \neg\,\mathbf{EG}\,\mathbf{EF}\,\neg p$. Also, for this Kripke structure we have $[\![\neg p]\!] = [\![q]\!]$. So, $[\![\mathbf{EG}\,\neg p]\!] = [\![\mathbf{EG}\, q]\!] = \{s_7\}$ and $[\![\mathbf{EF}\,\neg p]\!] = [\![\mathbf{EF}\, q]\!] = S \setminus \{s_3\}$.

By computing a smallest fixed point, we have $[\![\mathbf{EF}\,\mathbf{EG}\,\neg p]\!] = \{s_0, s_6, s_7\}$, and therefore $[\![\neg\,\mathbf{EF}\,\mathbf{EG}\,\neg p]\!] = \{s_1, s_2, s_3, s_4, s_5\}$.

By computing a largest fixed point, we have $[\![\mathbf{EG}\,\mathbf{EF}\,\neg p]\!] = S \setminus \{s_2, s_3\}$, and therefore $[\![\neg\,\mathbf{EG}\,\mathbf{EF}\,\neg p]\!] = \{s_2, s_3\}$.