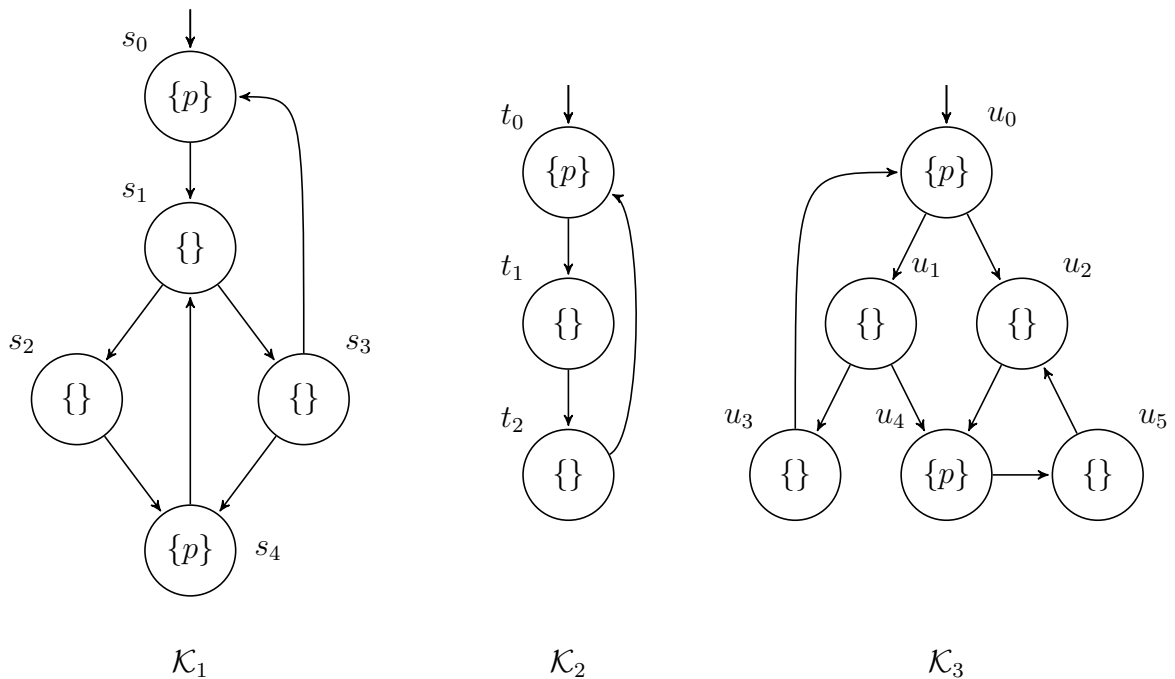# Model Checking – Exercise sheet 11

Exercise 11.1

Consider the following Kripke structures $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$, over $AP = \{p\}$:

(a) Does $\mathcal{K}_2$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

(b) Does $\mathcal{K}_2$ simulate $\mathcal{K}_3$? If yes, give a simulation relation. Otherwise, explain why.

(c) Does $\mathcal{K}_3$ simulate $\mathcal{K}_2$? If yes, give a simulation relation. Otherwise, explain why.

(d) Does $\mathcal{K}_3$ simulate $\mathcal{K}_1$? If yes, give a simulation relation. Otherwise, explain why.

Exercise 11.2

Let $\mathcal{K}_1$, $\mathcal{K}_2$, and $\mathcal{K}_3$ be Kripke structures. Show that if $\mathcal{K}_1$ and $\mathcal{K}_2$ are bisimilar, and $\mathcal{K}_2$ and $\mathcal{K}_3$ are bisimilar, then $\mathcal{K}_1$ and $\mathcal{K}_3$ are also bisimilar.

Exercise 11.3

Consider the following program with a Boolean variable x. Initially, the value of x is `false`. The question mark stands for a nondeterministic value.

```
1  x = ?;
2  while (x)
3    x = ?;
4  while (true) {}
```

Let $AP = \{x\}$, where x is true only in states where the variable x is `true`.

(a) Construct a Kripke structure $\mathcal{K} = (S, \rightarrow, r, AP, \nu)$ for the above program.

(b) Let $\approx$ be an equivalence relation on S such that for all $s \approx t$ we have $\nu(s) = \nu(t)$. Construct from $\mathcal{K}$ the abstracted Kripke structure $\mathcal{K}'$ w.r.t. $\approx$.

(c) Model check the following formulas with $\mathcal{K}'$. Refine the abstraction if necessary.

    (i) $\neg x \ \mathbf{W} \ x$

    (ii) $\mathbf{G}(\neg x \rightarrow \mathbf{X} \neg x)$

    (iii) $\mathbf{X}(\neg x \rightarrow \mathbf{G} \neg x)$