Technische Universität München **Summer Semester 2015**
I7
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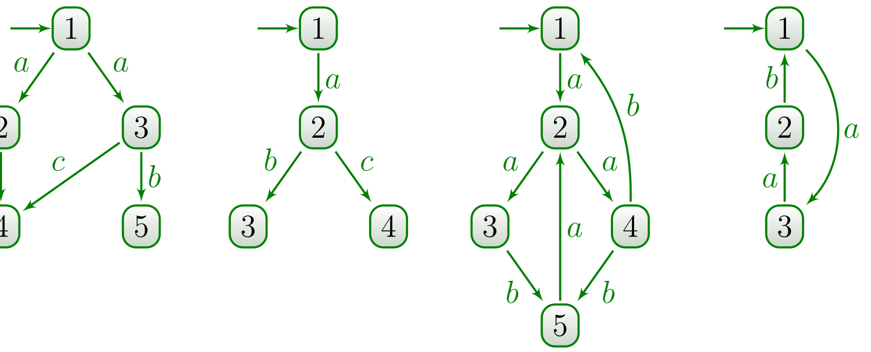
Model Checking – Exercise sheet 11

Exercise 11.1: Simulation and bisimulation

We consider the four following transitions systems (denoted T_1, T_2, T_3, T_4):



1. Does T_2 simulate T_1 ? If it is the case exhibit a simulation relation, if not, provide an counterexample.
2. Does T_1 simulate T_2 ? Exhibit a simulation relation or provide a counterexample.
3. Are T_1 and T_2 bisimilar ? Give a winning strategy for one of the players of the bisimulation game.
4. Are T_3 and T_4 bisimilar ? Give a winning strategy for one of the players of the bisimulation game.
5. Given three transitions systems S_1, S_2 and S_3. If S_1 and S_2 are bisimilar, and S_2 and S_3 are also bisimilar, prove that S_1 and S_3 are bisimilar.

Exercise 11.2: Abstraction of a simple program

We give the simple program over the integer variables `x`, `y`, `z`:

```
while(true) {
    z  = y;
    y  = y + x;
    x  = z;
}
```

1. What is the size (i.e. number of states) of the transition system of that program ?
2. We now omit the concrete values of the variables, and only retain the following information:
   - the control state of the program
   - predicates $(x > 0), (y > 0)$ and $(z > 0)$.

   Draw the resulting (abstract) Kripke structure. (18 states)

Exercise 11.3: Abstraction of a more complex program

We give the following program over the integer variables `x`, `y`, `z`, `x2`, `y2`:

```
while(true) {
    x2 = x + z;
    y2 = z + y;
    z  = x - y;
    x  = x2;
    y  = y2;
}
```

1. We omit the concrete values of the variables, and only retain the following information:
   - the control state of the program
   - predicates $(x > 0), (y > 0), (z > 0), (x2 > 0)$ and $(y2 > 0)$.

   In the abstract Kripke structure, from an initial state where all the predicates hold, can we reach a state where none hold ? If possible, give a trace.
2. In the concrete transition system, can we reach such a state ?
3. Exhibit a trace in the abstract Kripke structure satisfying the LTL property:
   $\mathbf{FG}(x > 0 \wedge y > 0 \wedge z > 0 \wedge x2 > 0 \wedge y2 > 0)$
4. Does the program admit a trace satisfying this property ?
5. Are the abstract Kripke structure and the transition system of the program bisimilar ? Justify your answer.