Technische Universität München **Summer Semester 2015**
I7
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Model Checking – Solution Sheet 8

Exercise 8.1: What is syntactic sugar ?

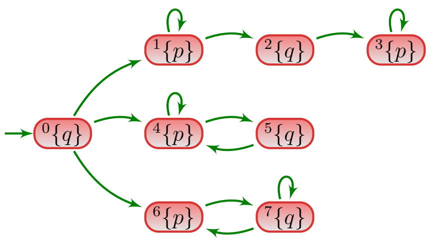
CTL operators are defined as QT where Q is A or E and T is any LTL modality $(\mathbf{X}, \mathbf{F}, \mathbf{G}, \mathcal{U}, \mathcal{W}, \mathcal{R})$. So many operators means a lot of cases to handle for inductively proving results on CTL.

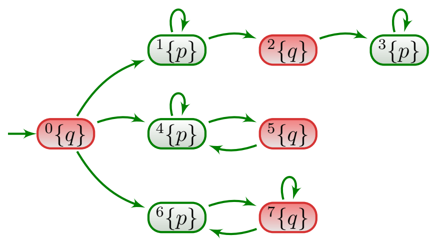
1. $A\mathbf{X}\varphi \equiv \neg E\mathbf{X}\neg\varphi$
   $E\mathbf{F}\varphi \equiv \text{true } E\,\mathcal{U}\,\varphi$
   $A\mathbf{G}\varphi \equiv \neg E\mathbf{F}\neg\varphi$
   $A\mathbf{F}\varphi \equiv \neg E\mathbf{G}\neg\varphi$
   $\varphi_1 E\,\mathcal{W}\,\varphi_2 \equiv E\mathbf{G}\varphi_1 \vee (\varphi_1 E\,\mathcal{U}\,\varphi_2)$
   $\varphi_1 A\,\mathcal{W}\,\varphi_2 \equiv \neg(\neg\varphi_2 E\,\mathcal{U}\,\neg(\varphi_1 \vee \varphi_2))$
   $\varphi_1 A\,\mathcal{U}\,\varphi_2 \equiv A\mathbf{F}\varphi_2 \wedge (\varphi_1 A\,\mathcal{W}\,\varphi_2))$
   $\varphi_1 E\,\mathcal{R}\,\varphi_2 \equiv \neg(\neg\varphi_1 A\,\mathcal{U}\,\neg\varphi_2)$
   $\varphi_1 A\,\mathcal{R}\,\varphi_2 \equiv \neg(\neg\varphi_1 E\,\mathcal{U}\,\neg\varphi_2)$

2. $E\mathbf{X}$ only provides a local information on the direct successors. $E\,\mathcal{U}$ only provides the existence of a branch which admits a treshold: the converse is the absence of a treshold on all branch. $E\mathbf{G}$ is thus necessary as it allows to state the absence of a treshold on a branch (or the presence of one on every branches).

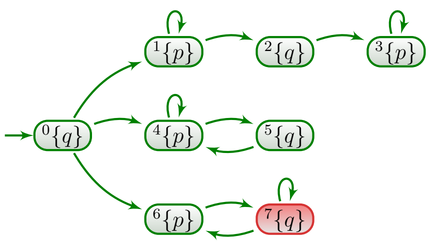
Exercise 8.3: Fixpoint computations

1. We recall that $E\mathbf{G}q$ is computed as the greatest fixpoint of the equantion $X = \mu(q) \cap pre(X)$. Thus we start by considering $X = [0, 7]$.

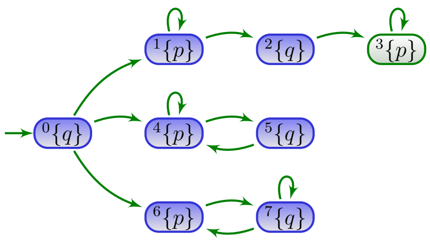




After the first iteration, since $pre([0, 7]) = [0, 7]$, we have all states at which q holds. Only 7 then has a predecessor for which q holds. The next iteration does not modify the set X computed, therefore follows states that validate $E\mathbf{G}q$.



$E\mathbf{F}q$ is computed as least fix point of the equation $X = \mu(q) \cup (pre(X))$, ie. initially $X = \emptyset$. After 3 iteration, we obtain the following fixpoint:



2. Remark that $A\mathbf{G}A\mathbf{F}p = \neg E\mathbf{F}E\mathbf{G}\neg p$. We start by computing $[\![E\mathbf{G}\neg p]\!]$. Remark that $[\![\neg p]\!] = [\![q]\!]$. Thus $[\![E\mathbf{G}\neg p]\!] = [\![E\mathbf{G}q]\!] = \{7\}$.

   Next we compute $[\![E\mathbf{F}\{7\}]\!]$, as the least fixpoint of $X = \mu(q) \cup (pre(X))$.

   $X_0 = \emptyset$, $pre(X_0) = \emptyset$, $\mu(\{7\}) = \{7\}$

   $X_1 = \{7\}$, $pre(\{7\}) = \{7, 6\}$, $\mu(\{7\}) = \{7\}$

   $X_2 = \{7, 6\}$, $pre(\{7, 6\}) = \{7, 6, 0\}$, $\mu(\{7\}) = \{7\}$

   $X_3 = \{7, 6, 0\}$, $pre(X_3) = X_3$

   $[\![E\mathbf{F}\{7\}]\!] = X_3 = \{0, 6, 7\}$

   $[\![\neg E\mathbf{F}\{7\}]\!] = \complement[\![\neg E\mathbf{F}\{7\}]\!] = \{1, 2, 3, 4, 5\}$

   Therefore $[\![A\mathbf{G}A\mathbf{F}p]\!] = \{1, 2, 3, 4, 5\}$

   Remark that $A\mathbf{F}A\mathbf{G}p = \neg E\mathbf{G}E\mathbf{F}\neg p$. We start by computing $[\![E\mathbf{F}\neg p]\!]$. Remark that $[\![\neg p]\!] = [\![q]\!]$. Thus $[\![E\mathbf{F}\neg p]\!] = [\![E\mathbf{F}q]\!] = \{0, 1, 2, 4, 5, 6, 7\}$.

   Next we compute $[\![E\mathbf{G}\{0, 1, 2, 4, 5, 6, 7\}]\!]$, as the greatest fixpoint of the equation $X = \mu(\{0, 1, 2, 4, 5, 6, 7\}) \cap pre(X)$

   $X_0 = [0, 7]$, $pre(X_0) = X_0$, $\mu(\{0, 1, 2, 4, 5, 6, 7\}) = \{0, 1, 2, 4, 5, 6, 7\}$, thus

   $X_1 = \{0, 1, 2, 4, 5, 6, 7\}$, $pre(X_1) = \{0, 1, 4, 5, 6, 7\}$, therefore

   $X_2 = \{0, 1, 4, 5, 6, 7\}$, $pre(X_2) = \{0, 1, 4, 5, 6, 7\}$,

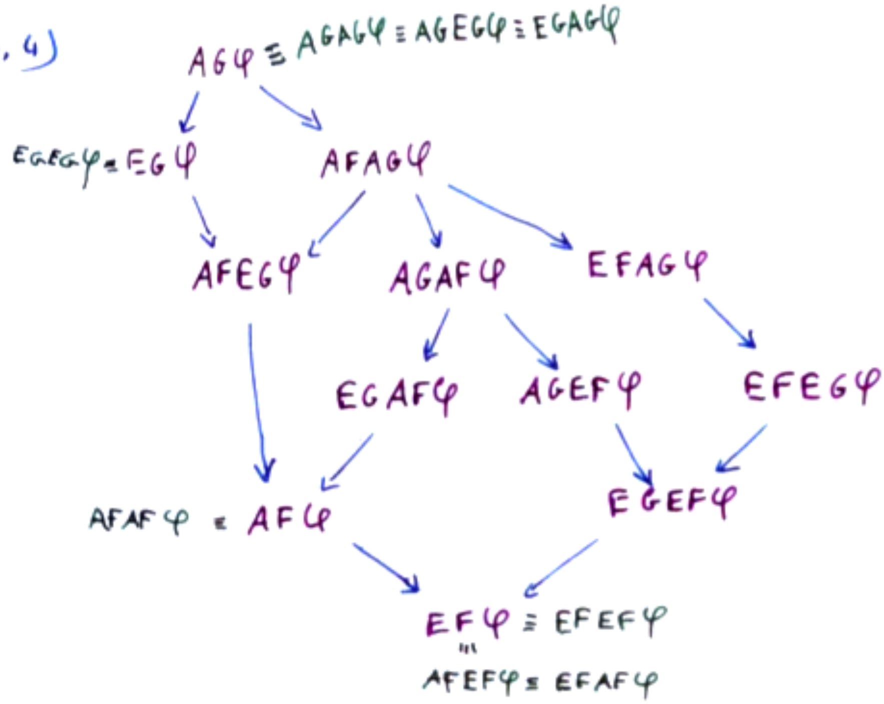
   so $[\![E\mathbf{G}\{0, 1, 2, 4, 5, 6, 7\}]\!] = X_2 = \{0, 1, 4, 5, 6, 7\}$. Therefore $[\![\neg E\mathbf{G}\{0, 1, 2, 4, 5, 6, 7\}]\!] = \complement\{0, 1, 4, 5, 6, 7\} = \{2, 3\}$.

   $[\![A\mathbf{F}A\mathbf{G}p]\!] = \{2, 3\}$

3. Remark the run $0, 6, 7, 7, \ldots$. Its trace is $\{q\}\{p\}\{q\}^\omega$ which validates neither $\mathbf{FG}p$, nor $\mathbf{GF}p$. Thus $K \not\models \mathbf{FG}p$ and $K \not\models \mathbf{GF}p$.

Ex 8.2

1, 2, 4)



3)

$\models EFp$, $\not\models AFp$

$\models AFp$, $\not\models EGp$

$\models EGp$, $\not\models AGp$