

Model Checking – Exercise sheet 7

Exercise 7.1: A simple partial order reduction

We consider a concurrent system \mathcal{S} consisting of 3 processes (p_1, p_2, p_3) .

Each process may access one local variable (l_1 for p_1 , l_2 for p_2 and l_3 for p_3) and may write values to a global variable g . Each process consists of a finite indexed sequence of instructions of the form:

- $(l_i == k)$ (tests whether the value of the local variable is k . If it is, executes the next instruction, else the instruction after that)
- $l_i := k$ or $l_i := l_i + k$ (assigns the corresponding value to the local variable, and then executes the next instruction)
- $g := \lambda l_i + k$ (assignment to the global variable, $\lambda, k \in \mathbb{N}$, then executes the next instruction)
- **goto** j (executes instruction j).

We define processes p_i ($1 \leq i \leq 3$) as follows (3 instructions):

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1  $l_i := i$ 
2  $g := l_i$ 
3 goto 1
```

1. Construct the transition system of process p_1 .
2. Give the set of states in the transition system of \mathcal{S} that are reachable in one or two steps.

We consider model-checking the following property over the value of the global variable: the smallest infinitely written value is even. Therefore, we take as set of atomic predicates $AP = \{p\}$: p holds when g has even value.

3. Propose a set of visible actions and an independence relation.

We consider a restriction on the Kripke structure of \mathcal{S} :

- If p_1 may execute an instruction other than a global variable assignment, it does

- else if p_2 may execute such an instruction, it does
- else if p_3 may execute such an instruction, it does
- otherwise the behaviour is the same as the original one.

4. Draw that restricted Kripke structure.

5. Does this restriction satisfy conditions c_0 to c_3 for these processes p_1, p_2, p_3 .

In the following we consider arbitrary processes but such that in any loop of any process, there is at least one assignment of the global variable.

6. Do conditions c_0 to c_3 hold in general for the restricted Kripke structure ?
7. If we allow (only) process p_2 to test the value of the global variable (through an action $g==k$), how to obtain a (non-trivial) partial order reduction ?
8. If we are interested in the model checking the same parity condition on the value of the local variable of p_1 , do we still have a partial order reduction ?
9. Propose a (non-trivial) partial order reduction.