Technische Universität München **Summer Semester 2015**
I7
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Model Checking – Exercise sheet 4

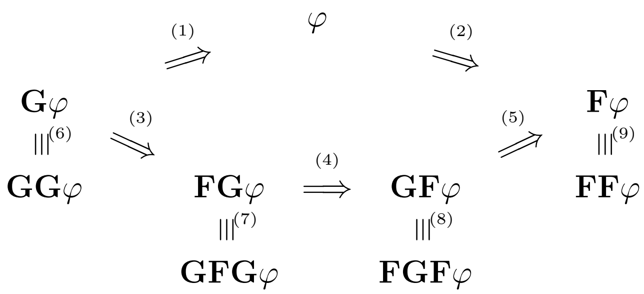
Exercise 4.1: Model checking

Let $AP = \{p, q, r, s\}$, which of the formulas $\varphi = \mathbf{G}\neg q \vee \mathbf{F}(q \wedge (\neg p\, \mathbf{W}\, s))$ and $\psi = \mathbf{G}((q \wedge \neg r \wedge \mathbf{F}r) \rightarrow ((p \rightarrow (\neg r\, \mathcal{U}(s \wedge \neg r)))\, \mathcal{U}\, r))$ hold for the following words:

1. $\{p,q\}\{p,q,r,s\}\{s\}\{p,q,r\}\{q,r,s\}\{p,q\}\{p\}\{\}\{p,q\}^\omega$
2. $\{p,q\}\{p,q,s\}\{s\}\{p,q,r\}\{q,r,s\}\{p,q\}\{p\}\{\}\{p,q\}^\omega$
3. $\{p,q\}\{q\}\{p,q,s\}\{p,q,s\}\{p,s\}\{q,r,s\}\{q,r\}\{q,r,s\}\{r,s\}\{q,r,s\}^\omega$
4. $\{p,q\}\{p,q,s\}\{p,r,s\}\{q,s\}\{p,s\}\{r,s\}\{r\}^\omega$
5. $(\{p\}\{s\}\{r\}\{q\})^\omega$

Exercise 4.2

We recall the following implications hold for any LTL φ:



(a) Using the definition of $\mathbf{F}$ and $\mathbf{G}$, and the semantics of $\mathcal{U}$, show (1) and (2)

(b) Deduce (3) and (5)

(c) Using the defintion of the semantics of $\mathbf{F}$, show that if $\varphi \implies \psi$ then $\mathbf{F}\varphi \implies \mathbf{F}\psi$

(d) Deduce if $\varphi \implies \psi$ then $\mathbf{G}\varphi \implies \mathbf{G}\psi$

(e) Deduce (4)

(f) Using the semantics of $\mathbf{F}$, show that $\mathbf{FF}\varphi \implies \mathbf{F}\varphi$

(g) Deduce (6)

(h) Deduce (9)

(i) Using the previously shown implications, deduce (7) and (8). Which one do you need for which direction ?

Exercise 4.3

A formula φ over the set of propositions **AP** is in *positive normal form* when the negations only appear directly in front of propositions $a \in \mathbf{AP}$. For instance $\neg \mathbf{G} a$ is not in positive normal form while $\mathbf{true}\ \mathcal{U} \neg a$ is.

We denote **NF-LTL** the set of formulas in positive normal form over the operators $\mathbf{X}, \mathcal{U}, \mathbf{G}, \wedge, \vee$ and $\neg$.

1. Show by induction, using the equivalences shown in the previous exercise sheet that any **LTL** formula φ admits an equivalent **NF-LTL** formula.
2. Let $\mathbf{NF\text{-}LTL}_{-\mathbf{G}}$ the formulas of **NF-LTL** in which the operator **G** does not occur. Show that for any formula φ in$\mathbf{NF\text{-}LTL}_{-\mathbf{G}}$, for any word $w \in \Sigma^\omega$, such that $w \models \varphi$ there exists an integer $N_\varphi(w)$ such that $w_0 \dots w_{N_\varphi(w)}$ characterizes whether $w \models \varphi$ or not. More formally for any $w' \in \Sigma^\omega$

$$w \models \varphi \iff w_0 \dots w_{N_\varphi(w)} w' \models \varphi$$

3. Let $\mathbf{NF\text{-}LTL}_{-\mathbf{X}}$ the set of **NF-LTL** in which **X** does not occur. Show that any $\mathbf{NF\text{-}LTL}_{-\mathbf{X}}$ formula φ can not distinguish $w \in \Sigma^\omega$ and $D(w) = w_0 w_0 w_1 w_1 \dots$, i.e.:

$$w \models \varphi \iff D(w) \models \varphi$$