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I7
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Model Checking – Exercise sheet 2

In this exercise sheet, we use, in addition to the modalities $\mathbf{X}$, $\mathcal{U}$, the following ones:

- **G**lobally: $w \models \mathbf{G}\varphi \iff w \models \neg(\mathbf{true}\, \mathcal{U}\, \neg\varphi)$
- Eventually (**F**inally): $w \models \mathbf{F}\varphi \iff w \models \neg\mathbf{G}\neg\varphi$
- **R**elease: $w \models \varphi\, \mathbf{R}\, \psi \iff w \models \neg(\neg\varphi\, \mathcal{U}\, \neg\psi)$

Exercise 2.1: Model checking

Given the set of atomic propositions $\{p, q, r, s\}$ and the LTL formulas $\varphi = \mathbf{GF}q$ and $\psi = \mathbf{G}((q \wedge \neg r \wedge \mathbf{F}r) \to (p \to (\neg r\, \mathcal{U}(s \wedge \neg r)))\, \mathcal{U}\, r)$, which of the following sequences validates those formulas:

- $\{p\}\{p\}\{p\}^\omega$, $\{q\}\{q\}\{q\}^\omega$, $\{s\}\{s\}\{s\}^\omega$, $\{q,r\}\{q,r\}\{q,r\}^\omega$ and $\emptyset\emptyset\emptyset^\omega$
- $\{r\}\{r\}\{q\}\{q\}(\{r\}\{q\})^\omega$, $\{r\}\{s\}\{r\}\{q\}\{q\}(\{r\}\{q\}\{q\})^\omega$ and $\{r\}\{r\}\{q\}\{s\}\{q\}(\{r\}\{r\}\{q\})^\omega$
- $rprqqq(rrrqqq)^\omega$ and $rprqqqs(rrrqqq)^\omega$ (p being a shortcut for $\{p\}$, similarly for q, r, s)
- $rrpqqqs(rrrqqq)^\omega$,$rrpqqsq(rrrqqq)^\omega$ and $rrpqqqrsr(qqrr)^\omega$
- $qqqrrpqqqqsqqq^\omega$ and $qqqrrpqpqpqqsqqrq^\omega$

Exercise 2.2: Specification

We give the following set of atomic propositions: $\{s, r, g\}$ standing for **s**ending a message, **r**eceiving a message, and **g**iving a result. Specify the following properties in **LTL**, and give an exemple satisfying it and another violating it.:

1. The process always returns a result
2. The process stops communicating after giving its result
3. The process emits infinitely many messages
4. The process only gives one answer
5. The process responds to each and every message
6. The process does nothing until it receives a message.

Exercise 2.3: Interpretation

Given the set of atomic propositions $\{p, q, r, s\}$, describe which properties the following formulas ensure. For each formula give an example of a word satisfying the formula and a counter example.

1. $p\,\mathcal{U}(q \vee \mathbf{G}q)$
2. $\mathbf{G}(q \rightarrow \mathbf{F}s)$
3. $\mathbf{G}((q \wedge \neg r \wedge \mathbf{F}r) \rightarrow (\neg r\,\mathcal{U}\,r))$
4. $p\,\mathcal{U}\,\mathbf{G}q$
5. $p\,\mathcal{U}\,\mathbf{F}q$
6. $\mathbf{G}(p\,\mathcal{U}\,\mathbf{G}q)$
7. $\mathbf{G}p\,\mathcal{U}\,\mathbf{G}q$

Exercise 2.4

Let $\Sigma = 2^{\mathbf{AP}}$ with $\mathbf{AP} \neq \emptyset$, and let $w \in \Sigma^\omega$, $w = w_0 w_1 \ldots$
We denote the i-th suffix of w as $w^i = w_i w_{i+1} \ldots$
Prove the following equivalences:

- $w \models \mathbf{G}\varphi \iff \forall i \in \mathbb{N}\ w^i \models \varphi$
- $w \models \varphi\,\mathbf{R}\,\psi \iff \forall i \in \mathbb{N}\ (\forall j < i\ w^j \not\models \varphi) \rightarrow w^i \models \psi$
- $w \models \neg(\varphi\,\mathcal{U}\,\psi) \iff w \models \neg\varphi\,\mathbf{R}\,\neg\psi$
- $w \models \varphi\,\mathcal{U}\,\psi \iff w \models \psi \vee (\varphi \wedge \mathbf{X}(\varphi\,\mathcal{U}\,\psi))$
- $w \models \varphi\,\mathbf{R}\,\psi \iff w \models \mathbf{G}\psi \vee (\psi\,\mathcal{U}(\varphi \wedge \psi))$

Exercise 2.5

Let $K_1 = (S, \rightarrow_1, r, \mathbf{AP}, v)$ and $K_2 = (S, \rightarrow_2, r, \mathbf{AP}, v)$ two Kripke structures with same set of state S, same initial state r and same interpretation of the predicates in $\mathbf{AP}$. We write $K_1 \leq K_2$ if the transition relation $\rightarrow_2 \subseteq S \times S$ contains the transition relation $\rightarrow_1$, that is $\rightarrow_1 \subseteq \rightarrow_2$.

Show that if $K_1 \leq K_2$ then for any **LTL** formula φ:

$$K_2 \models \varphi \implies K_1 \models \varphi$$