## Model Checking - Exercise sheet 7

## Exercise 7.1: Some BDD's

Let $a_{0} a_{1} a_{2} \ldots a_{n}, b_{0} b_{1} \ldots b_{n}$ and $c_{0} c_{1} \ldots c_{n+1}$ three integers written in binary least significant digit first.

- Give an order over the variables and a BDD such that $a+b=c$
- Give an order over the variables and a BDD such that $c=3 b$


## Exercise 7.2: Simulation and Bisimulation

- Let $K_{1}$ and $K_{2}$ two Kripke structures, assume $K_{1} \subseteq K_{2}$ (i.e. for any state $q$ of $K_{1}, q$ is also a state in $K_{2}$, and for any transition $\left(q_{1}, q_{2}\right)$ in $K_{1},\left(q_{1}, q_{2}\right)$ is also a transition in $K_{2}$. Does one structure simulate the other ? Are they bisimilar?
- Are the two following Kripke structures bisimilar:

- TODO: find two Kripke structures, with labels on nodes and not on transitions such that the first one (untrivially) simulates the second one.


## Exercise 7.3: Abstraction

We will study in this exercise some simple C programs over two integer variables X and Y . We are interested in the sign of those variables, we therefore define two predicates $p$ and $q$ which holds when the value of X (resp. Y) is postive.

```
X = X + X
Y = Y - X
Y = Y - X
Y = Y + X
Y = Y + Y
X = X - X
```

For this program, we not only have the predicates $p$ and $q$ but also the control point (i.e. line number).

- Assume we have: $i \quad \mathrm{X}=\mathrm{X}+\mathrm{Y}$

What are the successors of $(i,\{p, q\})$ ? What are the successors of $(i,\{q\})$ ?

- Build the transition system from $(1,\{p, q\})$ using this step by step construction.
- What happens in practice?

```
while (X != Y)
    if (X < Y)
        Y = Y - X;
    else
        X = X - Y;
return X;
```

- Build the transition system for that program from $(0,\{p, q\})$
- Build the transition system when introducing another predicate $r$ which holds when $X<Y$

