Model Checking – Exercise sheet 5.1

Exercise 5.1: $a\,\mathcal{U}\,(b\,\mathcal{U}\,c)$ **and** $(a\,\mathcal{U}\,b)\,\mathcal{U}\,c$

Let $\varphi_1 = (p \wedge q)\,\mathcal{U}\,((p \wedge \neg q)\,\mathcal{U}\,(\neg p \wedge q))$ and $\varphi_2 = ((p \wedge q)\,\mathcal{U}\,(p \wedge \neg q))\,\mathcal{U}\,(\neg p \wedge q)$ two LTL formulas over the set of propositional variables $\{p, q\}$.

1. Are these LTL formulas equivalent ?
2. We denote $a = \{p, q\}, b = \{p\}, c = \{q\}, d = \emptyset$. Write ω-regular expression E_1 and E_2 that describes infinite words over alphabet $\{a, b, c\}$ that validate φ_1 and φ_2.
3. Give a Büchi automaton accepting each of those languages.
4. Given a Büchi automaton $\mathcal{A} = (Q, Q_0, F, \Delta)$ over $\Sigma = \{a, b, c\}$. We denote $L_q^{\mathcal{A}}$ the language of infinite words accepted starting from state q in the automaton $\mathcal{A}$. Which of the following statements hold:
   - If $L_{q_1} \subseteq L_{q_2}$, $(q_3, \alpha, q_1) \in \Delta$ and $(q_3, \alpha, q_2) \in \Delta$, then removing (q_3, α, q_1) does not modify the language accepted by $\mathcal{A}$.
   - If $L_{q_1} \subseteq L_{q_2}$, $(q_3, \alpha, q_1) \in \Delta$ and $(q_3, \alpha, q_2) \in \Delta$, and q_1 is not an accepting state, then removing (q_3, α, q_2) does not modify the language accepted by $\mathcal{A}$.
   - There exists a set of transitions of $\mathcal{A}$ (i.e. a subset of Δ), such that a word w is in $L(\mathcal{A})$ iff there exists a run of w in $\mathcal{A}$ that fires infinitely often one of the transitions in that set.
   - If a state q has no successor, then it can be removed, and all transitions mapping to that state can also be removed. This process can be iterated without modifying the language.
5. We will now apply the LTL to Büchi translation for those formulas. In order to obtain more concise automata, we suppress $(\neg p \wedge \neg q)$ transitions (we disallow d). Describe the set of states.
6. Assume φ_1 is present in one state: what is required for this state to have an outgoing transition ?

7. Assume φ_1 is not present in one state (i.e. $\neg\varphi_1$) is present in that state: what is required for this state to have an outgoing transition ?

8. Same questions for φ_2 and other non-trivial subformulas of φ_1 and φ_2.

9. Which states in these automata do not have any outgoing transition ?

10. Build the automata over Σ, translation of φ_1 and φ_2.

Exercise 5.2: $\mathbf{G}(a \to ((a \vee b)\ \mathcal{U}\ c))$

Using the previous exercise notation, we consider the LTL formula $\psi = \mathbf{G}(a \to ((a \vee b)\ \mathcal{U}\ c))$.

1. Which are the subformulas of ψ ?

2. Prove the following claim:
   states which do not imply formula $a \to ((a \vee b)\ \mathcal{U}\ c))$ can be removed.

3. Can we perform a similar trick when translating an **F** formula ?

4. Build the Büchi automaton accepting the language described by ψ.