

## Model Checking – Exercise sheet 4

### Exercise 4.1

We give the following regular expressions over defining regular languages over the alphabet  $\Sigma = \{a, b\}$  ( $C$  denotes the complement w.r.t.  $\Sigma^*$ ).

$$L_1 = a^*(baa^*)ba^* \quad L_2 = (a^*ba)^*a^*ba^* \quad L_3 = C(\Sigma^*bb\Sigma^*)$$

1. Which of those regular expressions denote the same language ?
2. Which is (are) the regular expression(s) accepted by the smallest deterministic automaton ?

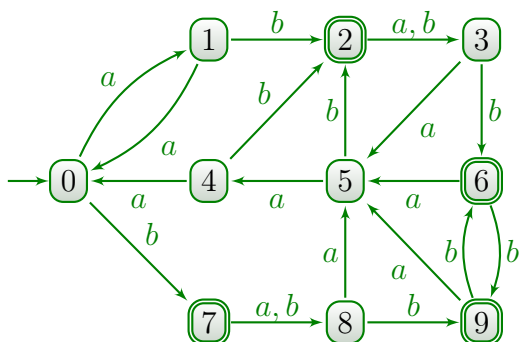
### Exercise 4.2

Give a regular expression for each of the following languages over alphabet  $\Sigma = \{a, b\}$ :

1. words that start with  $b$  and don't contain pattern  $ab$
2. words in which any two  $b$  are always separated by at least one  $a$
3. words that do not contain the pattern  $aab$
4. words containing either two  $a$  or three  $b$
5. words with as many  $a$  than  $b$
6. words in which at each position the number of preceding  $a$  is the same as the number of preceding  $b$  up to a difference of 3.
7. words in which the pattern  $ab$  occurs as often as the pattern  $ba$

### Exercise 4.3: Minimization

Minimize the following automaton:



### Exercise 4.4: True or False

Prove or disprove each of the following assertions:

1. Let  $A_1 = (q_0, Q_1, \delta_1, F_1)$  and  $A_2 = (q'_0, Q_2, \delta_2, F_2)$  two deterministic finite automata. The minimal automaton accepting  $\mathcal{L}(A_1) \cup \mathcal{L}(A_2)$  may have  $|Q_1| \cdot |Q_2|$  states.
2. For any non-deterministic automaton, one can build a deterministic automaton accepting the same language.
3. The minimal deterministic automaton of a non deterministic automaton  $A = (Q_0, Q, \Delta, F)$  may have a number of states exponential w.r.t.  $Q$ .
4. The reverse language of a regular language is regular.
5. It suffices to reverse the transition and change accepting for initial and initial to accepting on an automaton to accept the reverse of the language accepted by an automaton.
6. To test whether two automata accept the same language we can test whether the two reverse automata accept the same language.
7. To test whether two deterministic automata accept the same language, we can test whether determinizing the two reverse automata lead to the same automaton.
8. The complement of a regular language is a regular language. It suffices make every accepting state non-accepting and vice-versa to build an automaton accepting the complement of that language.
9. Any regular language is accepted by a minimal deterministic automaton.
10. Any regular language is accepted by a minimal non-deterministic automaton.
11. Any regular language admits a minimal regular expression.

### Exercise 4.5: And for infinite words

Prove or disprove propositions 1,2,3,8,9,10 for infinite words.