I. Higher-order programs. ...

II. LTL. If the webpage does not display properly in your browser, we have prepared a PDF file for you. Do not be intimidated by the number of tasks: with a single exception, each task is trivial.

A. Let AP = {green, yellow, red} and σ = ({green}{green}{green}{yellow}{red}{red}{red}{yellow,red})$^\omega$. Does σ satisfy the following properties:
1. $\bigcirc$green ∨ yellow ,
2. ¬green**U**red ,
3. ¬(green**U**red) ?

B. Let AP = {green, yellow, red}. Write the following properties as LTL formulas (derived operators are allowed):
1. Red and yellow occur together infinitely often.
2. From some time point onward red and green never occur together.
3. Whenever green turns on, green continues for at least two consecutive time units.

C. Consider a program P with State={s_0, s_1, s_2}, init={s_0}, transitions $s_1 \to s_0 \to s_2 \to s_1$. Let AP={$a$,$b$}. Let a label just s_1 and b label just s_2. Which of the following formulas hold for P?
1. $\Diamond a \wedge \Diamond b$,
2. $\Diamond(a \wedge b)$,
3. $\Diamond$a **U** $\Box\neg$(a∧b) ,
4. $\Box\Diamond$b .

D. Let AP be a set of atomic propositions and φ,ψ be LTL formulas over AP. Show the following properties about distributivity, negation propagation, and expansion of temporal connectives:
1. $\bigcirc(\varphi\wedge\psi) \equiv \bigcirc\varphi\wedge\bigcirc\psi$,
2. $\bigcirc(\varphi\mathbf{U}\psi) \equiv \bigcirc\varphi\mathbf{U}\bigcirc\psi$,
3. $\neg\Box\varphi \equiv \Diamond\neg\varphi$,
4. $\neg(\varphi\mathbf{U}\psi) \equiv \neg\varphi\mathbf{R}\neg\psi$,
5. $\neg(\varphi\mathbf{W}\psi) \equiv (\varphi\wedge\neg\psi)\mathbf{U}(\neg\varphi\wedge\neg\psi)$,
6. $\neg(\varphi\mathbf{R}\psi) \equiv \neg\varphi\mathbf{U}\neg\psi$,
7. $\varphi\mathbf{U}\psi \equiv \psi\vee(\varphi\wedge\bigcirc(\varphi\mathbf{U}\psi))$,
8. $\varphi\mathbf{W}\psi \equiv \psi\vee(\varphi\wedge\bigcirc(\varphi\mathbf{W}\psi))$.

E. Let AP={green, yellow, red}. Convert the following formulas into positive LTL:
1. ¬((yellow**U**green)**U**red) ,
2. ¬(green**W**(red**U**green)) ,
3. ¬((yellow**U**green)**R**(red**U**green)) .

F. (A task with an increased level of difficulty, **.) Show that weak until is "the greatest solution of the expansion law". More formally, show that for all LTL formulas φ,ψ over a set of atomic propositions AP,
- Words(φ**W**ψ) is a fixpoint of the map $\lambda\ S\in\wp(\mathbb{N}_0\to\wp(AP)).$ Words(ψ) ∪ {σ∈Words(φ) | σ[1..]∈S}
- and that it is the greatest of all such fixpoints.