Model Checking, SS2011: Exercise Sheet 14

July 5, 2011

Exercise 14.1. In this exercise we consider a variant *LTL'* of the logic LTL presented in class. Let

1. Σ be a set of atomic propositions denoted by p,
2. *Formulas* be the set of LTL formulas,
3. $V : \mathbb{N} \to 2^{\Sigma}$ be a valuation of atomic propositions,
4. $\models \subseteq (\mathbb{N} \to 2^{\Sigma}) \times \mathbb{N} \times \mathit{Formulas}$ be the *LTL'* satisfaction relation defined like the satisfaction relation for LTL except for the following case.

$$(V, i) \models p \textit{ iff } p \in V(i)$$

Consider the following Prolog facts.

```
v(1, [0-[p1], 1-[p1,p2,p3], 2-[p1], 3-[p2]]).
v(2, [0-[p1,p2], 1-[p2,p3], 2-[p2], 3-[p2]]).
v(3, [0-[p1,p2], 1-[p2,p3], 2-[p2,p3], 3-[p2]]).
```

A fact `v(Id, V)` contains the user defined valuation $\mathtt{V} : \mathbb{N} \to 2^{\Sigma}$ number `Id`. Define a Prolog procedure `s/3` such that `s(V, I, F)` succeeds if `F` is a formula with no occurrences of the $\mathcal{U}$, $\Box$ or $\diamond$ modal operators, and $(\mathtt{V}, \mathtt{I}) \models \mathtt{F}$. Give your answer by completing the following clauses.

```
s(V, I, P) :-
s(V, I, F/\G) :-
s(V, I, not(F)) :-
s(V, I, next(F)) :-
s(V, I, F\/G) :-
s(V, I, F>>G) :-
```

Exercise 14.2. Consider the following alphabet $\Sigma = \{a, b, c\}$. For each of the following properties on words over Σ construct a corresponding monitor and ω-regular expression.

1. The word α has at least one letter c.

2. In the word α every letter a is immediately followed by letter b.

3. In the word α there are at least two b's in between two successive a's.

4. In α, has a suffix of a's.

5. In α, b occurs infinitely many often.

6. In α, b always eventually follows a.

7. In α, there are finitely many occurrences of a.

Exercise 14.3. Consider the following set of LTL' atomic propositions $\Sigma = \{a, b, c\}$. For each of the following properties on valuations LTL' give an LTL' formula satisfied by correct valuations. Formally, for each property $P(V)$ over a valuation variable V, give a formula ϕ such that $\forall\ V\ .\ P(V) \rightarrow ((V, 0) \models \phi)$.

1. The valuation V assigns true to c at least once.

2. In the valuation V proposition a is immediately followed by proposition b.

3. In valuation V there are at least two b's in between two successive a's.

4. In V eventually a always holds.

5. In V, b holds infinitely many often.

6. In V, b always eventually follows $a \wedge c$.

7. In V, a is true finitely many times.

Exercise 14.4. Go to `http://buchi.im.ntu.edu.tw/` and look for Büchi automata corresponding to the answers you gave for Exercise 14.3.

Exercise 14.5. Let $A(\phi)$ be the set of propositions occurring in LTL formula ϕ.

1. Give a set of defining equations for $A(\phi)$.

2. Let V be an LTL' valuation and ϕ an LTL formula. Let V' be the restriction of V to the propositions occurring in ϕ, i.e. let V' be such that $\forall\ i \in \mathbb{N}\ .\ V'(i) = V(i) \cap A(\phi)$. Prove that

$$\forall\ i \in \mathbb{N}\ .\ \big((V, i) \models \phi \text{ iff } (V', i) \models \phi\big)$$