Model Checking, SS2011: Exercise Sheet 6

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Note. Let α denote the abstraction function *more*.

Exercise 6.1. Consider the definition of the relational composition operator

$$\varphi(v, v') \circ \psi(v, v') := \exists v'' \,.\, \varphi(v, v'') \wedge \psi(v'', v')$$

Consider the transition relations $\rho_1, \ \ldots, \ \rho_5$ in exercise 2.6. Compute the following relational compositions.

1. $\rho_1 \circ \rho_2$
2. $\rho_3 \circ \rho_5$
3. $\rho_3 \circ \rho_4$
4. $\rho_1 \circ \rho_4$

Exercise 6.2. Prove that

$$post(\psi, post(\varphi, \delta)) \equiv post(\varphi \circ \psi, \delta)$$

Exercise 6.3. Let $p \in Preds$. Let α denote the abstraction function *more*. Prove that for all φ such that $\varphi \models p$, it holds that $\alpha(\varphi) \models p$.

Exercise 6.4. Consider the program P from exercise 2.6 and the set of predicates $Preds = \{pc = l_1, \ \ldots, \ l_5, \ \bot\}$. Let I and J be predicates satisfying the (concrete reachability) proposition

$$post(\rho_1, pc = l_1) \models I \wedge post(\rho_3, I) \models J \wedge post(\rho_5, J) \models \bot$$

Prove that giving α the set of predicates $Preds' = Preds \cup \{I, J\}$, I and J satisfy the (abstract reachability) proposition

$$\alpha(post(\rho_1, pc = l_1)) \models I \wedge \alpha(post(\rho_3, I)) \models J \wedge \alpha(post(\rho_5, J)) \models \bot$$