

Model-Checking SS 2010

Exercise 8

July 13

Due date: July 20, 9:55am CET.

1. Give two generic instances of $\forall\alpha.(\alpha \rightarrow \alpha) \rightarrow \alpha$
2. Prove or give a counterexample for the following statement:

If τ , τ_1 and τ_2 are types, α and β are distinct type variables, then $[\tau_2/\beta][\tau_1/\alpha]\tau = [\tau_2/\beta, \tau_1/\alpha]\tau$

3. For each expression e in the following list:

- Show how algorithm W computes a well-type of e ; show this by constructing a tree of calls to W that reflects the operations performed by W, and by showing how each m.g.u. is computed.
- Given the well-type τ of e obtained in the last step, construct a derivation of the well-typed judgement $\emptyset \vdash e : \tau$.

(a) $\lambda a \lambda b. b - a$

(b) let $d = \lambda a \lambda b. b - a$ in $d \ 1 \ 2$

(c) $\lambda x. \text{if } x \text{ false true}$

assume there exists a constant $\text{if} : \forall\alpha. \text{bool} \rightarrow \alpha \rightarrow \alpha \rightarrow \alpha$

(d) $\lambda m \lambda x \lambda y.$

let $d = \lambda a \lambda b. b - a$ in

$\text{if}_u ((d \ x \ y) = 0) (\text{print } m)$

assume there exist constants $\text{if}_u : \text{bool} \rightarrow \text{unit} \rightarrow \text{unit}$ and $\text{print} : \text{string} \rightarrow \text{unit}$

(e) $\lambda f. f \ 1$

(f) $\lambda x. \text{let } \text{fact} =$

$\text{fix } (\lambda \text{fact} \lambda x. \text{if } (x \leq 0) \ 0 \ (n * (\text{fact}(x - 1))))$

in $\text{fact } x$

assume there exists a constant $\text{fix} : \forall\alpha. (\alpha \rightarrow \alpha) \rightarrow \alpha$

4. The following expressions cannot be well-typed. Show how W would attempt to construct a well-type for each expression (i.e. construct a tree of calls to W that reflect the operations performed by W , and show how each m.g.u. is computed) and pinpoint the reason for W 's failure.

(a) $(\lambda i.ii)(\lambda x.x)$

(b) $\lambda f.\text{let } x = f\ 1 \text{ in } f\ \text{true}$