Logic ST 2014

http://tinyurl.com/logic2014

- Lecturer: Stefan Göller
  - room: MI 03.11.044
  - e-mail: stefan.goeller@gmail.com
- Dates and rooms:
  - each Tuesday **lecture** 8-10, room 00.04.011, MI Hörsaal 2 (5604.EG.011)
  - each Wednesday **lecture** 12-14, room 00.06.011, MI Hörsaal 3 (5606.EG.011)
  - each Thursday **exercise** 14-16, 03.09.014, Labor CIM (5609.03.014)
- Format: (6SWS=4+2)

Logic ST 2014 (II)

- **Examination**: Written exam (2 hours)
- active participation by presentations and discussions in the exercises
- Please jot down examples, proofs, explanations, etc presented in the lecture
- Literature will be linked on course website

4				9				2
		1				5		
	9		3	4	5		1	
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Sudoku

4				9				2
		1				5		
	9		3	4	5		1	
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Fill the empty squares so that every row, column, and 3x3 block contains all digits from 1 to 9.

Why 6?

4				9				2
		1				5		
	9		3	4	5		1	6
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Why 6?

4				9				2
		1				5		
	9		3	4	5		1	6 7
		8				2	5	3 7
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	3 7
		9				6		
5				7				4

A Very Brief History of Logic

Origin in Ancient Greece: Aristotle (384–322 BC) studies the nature of arguments and deduction.

Several works, e.g. *Analytica priora*, *Analytica posteriora*.

Systematic study of syllogisms.

Syllogisms

A syllogism (Greek: syllogismos conclusion, inference) is a kind of logical argument that applies deductive reasoning to arrive at a conclusion based on two or more propositions that are asserted or assumed to be true.

Examples:

If all men are mortal and
Socrates is a man,
then Socrates is mortal.

If a number is even and bigger than two,
then it is not a prime.

If interest rates are high,
then stockbrokers are unhappy.

Syllogisms: reinterpreting Aristotle

A syllogism is discourse in which, certain things being stated, and even assuming that we only know the meaning of "if, then, and, is (are), all, some, none", something other than what is stated follows of necessity from their being so.

If all X are Y and
S is X,
then S is Y.

If X is Y and Z,
then X is not P.

If X are Y,
then Z are W.

Syllogisms

Aristotle compiled a list of the valid syllogisms.

All spaniels are dogs	All P are M	
All dogs are animals	All M are S	(Barbara)
All spaniels are animals	All P are S	

No flower is an animal	No P is M	
Alle dogs are animals	All S are M	(Cesare)
No flower is a dog	No P is S	

All dolphins live in the see	All M are P	
All dolphins are mammals	All M are S	(Darapti)
Some mammals live in the see	Some S are P	

Criticism (with hindsight)

Many valid deductions are not contained in Aristotle's list, e.g.

$$\frac{\text{Some cat is feared by every mouse}}{\text{All mice fear at least one cat}} \qquad \frac{\text{Some A are B}}{\text{All C are D}}$$

Aristotle does not provide any calculus to handle long chains of deductions.

Propositional logic (I)

George Boole (1815 – 1864)

Atomic propositions can be true or false (we do not know which!)
Linked using operators (and, or, not, if-then ...).
No quantification (all, some).

Example:

- Propositions: "Alice is an architect", "Bob is a lawyer".
- Four possible situations or worlds:
  (1) Alice is an architect, Bob is a lawyer.
  (2) Alice is an architect, Bob is not a lawyer.
  (3) Alice is not an architect, Bob is a lawyer.
  (4) Alice is not an architect, Bob is not a lawyer.

Propositional logic (II)

- Some of the possible combined propositions:
  "Alice is an architect and Bob is a lawyer".
  "If Alice is an architect then Bob is a lawyer".
  "If Alice is not an architect, then Bob is not a lawyer".
  "If Bob is not a lawyer, then Alice is not an architect".

B is a consequence of A: B is true in all worlds in which A is true.

Algebraic calculus to decide whether B is a consequence of A.
The calculus is based on the analogy between true and 1, false and 0, or and addition, and and multiplication.

Predicate logic (I)

Frege, Peano, Russell (end of the 19th century)

Logic as foundation of mathematics, as formal tool for avoiding contradictions.

Development of predicate logic, which allows to

- describe relations between entities.
- formulate existential statements: "there is an x such that ... holds".
- formulate universal statements: "for all x ... holds".

If there exists a cat that is feared by every mouse then
for every mouse there exists a cat that the mouse fears

Predicate logic (II)

Takes a central role in the foundations of mathematics.

Classical example: The Peano axioms for the natural numbers.

- $0 \in \mathbb{N}$
- $\forall n \in \mathbb{N}\ \exists n' \in \mathbb{N} : n' = \mathsf{succ}(n)$
- $\forall n \in \mathbb{N} : \mathsf{succ}(n) \neq 0$
- $\forall n, m \in \mathbb{N} : (\mathsf{succ}(n) = \mathsf{succ}(m) \ \rightarrow \ n = m)$
- $\forall X \subseteq \mathbb{N} : (0 \in X \wedge \forall n \in \mathbb{N} : (n \in X \ \rightarrow \ \mathsf{succ}(n) \in X)) \rightarrow \ \mathbb{N} \subseteq X$

Logic in Computer Science

Shannon (1916 – 2001) shows in 1937 that propositional logic (boolean algebra) can be used to describe and optimize electromechanical circuits.

Newell, Simon, Robinson develop in 1950-1960 the first systems for the mechanization of logical deduction, and apply them to artificial intelligence problems.

Applications in computer science

- Digital circuit design: digital circuits can be described as logical formulas $\leadsto$ Circuit design and optimization
- Models and specifications: Precise description of complex systems and requirements
- Verification: proving that a program satisfies its specification
- Databases: Formalization of queries $\leadsto$ Query language SQL (Structured query language)
- Complexity theory: Logical characterizations of complexity classes (descriptive complexity)
- Artificial Intelligence:
  - General game playing
  - Theorem provers: automated or computer assisted mathematical proofs $\leadsto$ automatic proof of important theorems in boolean algebras, four-color theorem,...

Problems of natural languages (I)

Problem: Assigning truth values to natural language sentences is difficult.

Examples:

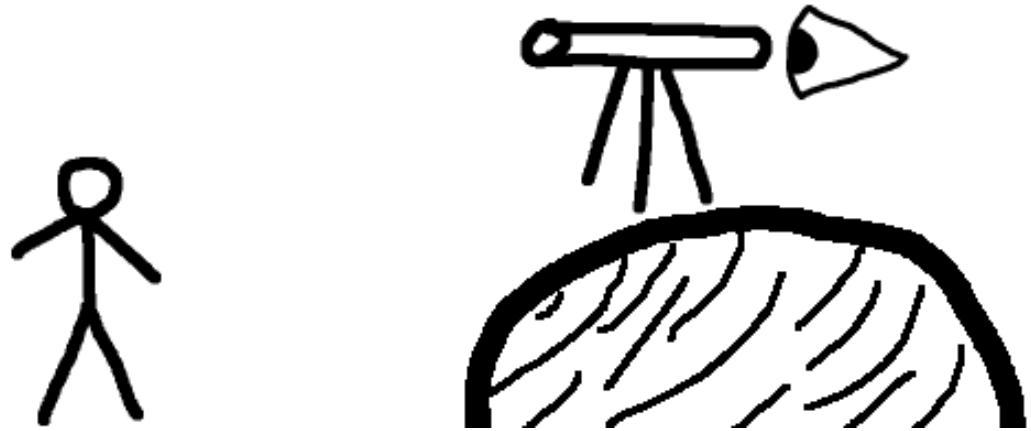
- I have only drunk a little.
- His money went up in smoke.
- Revenge is a dish best served cold.

Problems of natural language (II)

Problem: Natural languages are ambiguous.

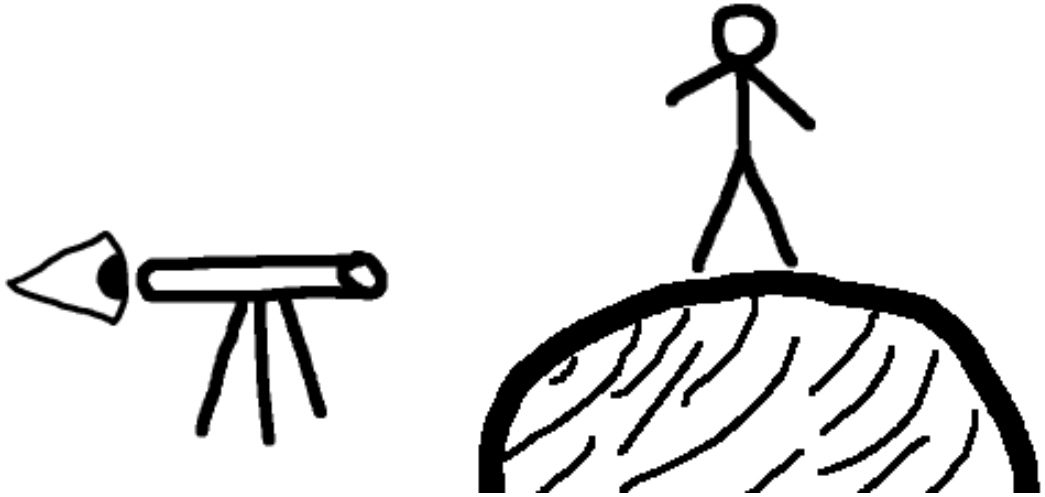
Example: I saw the man on the hill with the telescope.

I saw the man . . .



(((I saw the man) on the hill) with the telescope)

I saw the man . . .

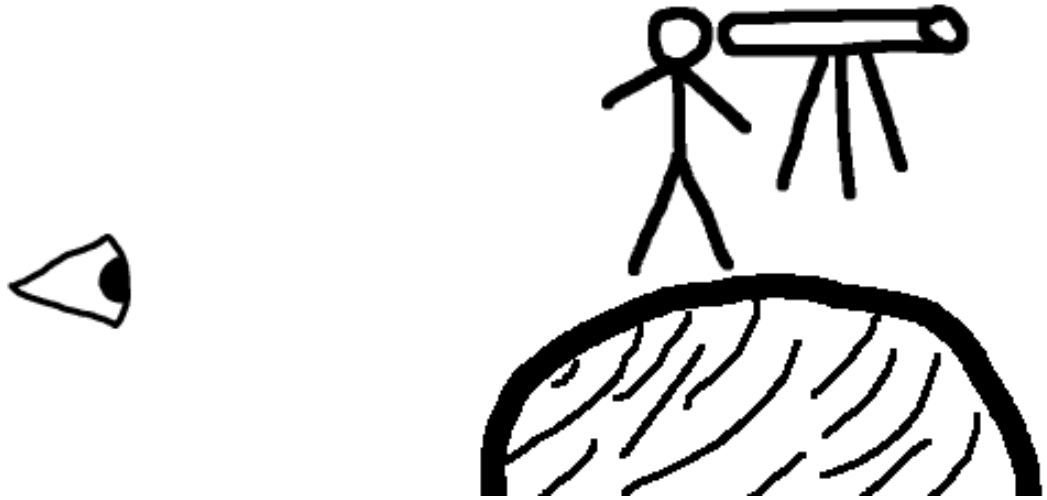


((I saw (the man on the hill)) with the telescope)

I saw the man . . .

((I saw the man) (on the hill with the telescope))

I saw the man . . .



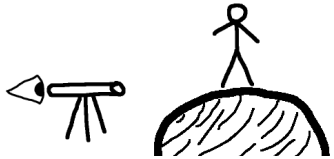
(I saw ((the man on the hill) with the telescope))

I saw the man . . .

(I saw (the man (on the hill with the telescope)))

I saw the man . . .

(((I saw the man) on the hill) with the telescope)



((I saw (the man on the hill)) with the telescope)

((I saw the man) (on the hill with the telescope))

(I saw ((the man on the hill) with the telescope))

(I saw (the man (on the hill with the telescope)))

5 possible interpretations

Problems of natural languages (IV)

Problem: Natural languages are not "context-free".

The Beatles are musicians
Paul McCartney is a Beatle

Paul McCartney is a musician

The Beatles are four
Paul McCartney is a Beatle

Paul McCartney is four