Technische Universität München
I7
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Logic

Exercise Sheet 9

Discussion: July 10, 2014

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1. Give a family of formulas $(F_k)_{k\geq 0}$ of predicate logic over the signature $S = \{E\}$, where E is a binary symbol such that the following holds for each $k \geq 0$:

- $\mathsf{qr}(F_k) = k$,
- $F_k = F_k(x, y)$, i.e. F_k has two free variables, and
- for each suitable structure $\mathcal{A}$ and each $a, b \in U_\mathcal{A}$ we have $\mathcal{A}_{[x/a,y/b]} \models F_k$ if and only if there is E-path from a to b of length k.

Construct a similar family $(F_k)_{k\geq 0}$ but where

- for each suitable structure $\mathcal{A}$ and each $a, b \in U_\mathcal{A}$ we have $\mathcal{A}_{[x/a,y/b]} \models F_k$ if and only if there is E-path from a to b of length 2^k.

2. Prove the second part of the induction proof for showing connectivity is not expressible in predicate logic. More precisely, consider the case $a \notin N_{2^{k-(i+1)}}(a_h)$ for each $h \in \{1, \ldots, i\}$.

3. Show that there is no formula in predicate logic expressing the transitive closure of a binary relation R.

4. Show the following equivalence for each subset $X \subseteq \mathbb{N}$ and each signature S:

(1) Property $\{\mathcal{A} : |U_\mathcal{A}| \in X\}$ is expressible in predicate logic with equality over signature S.

(2) X or $\mathbb{N} \setminus X$ is finite.