Technische Universität München
I7
Stefan Göller ST 2014

Logic

Exercise Sheet 8

Discussion: June 26, 2014

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1. Which of the following problems are decidable?

a) Given a formula F of predicate logic, does F have an infinite model?

b) Given a formula F of predicate logic without equality and without any functional symbols and with only one unary relational symbol, say R, is F satisfiable?

c) Given two formulas F and G of predicate logic, is every structure that is suitable for F and G a model of precisely one of those two formulas?

d) Given a formula F of predicate logic, does F have at least three different models?

e) Given a formula F of predicate logic in prenex form without any universal quantifiers, is F satisfiable?

2. Prove or refute:

a) If T_1 and T_2 are theories, then $T_1 \cup T_2$ is a theory.

b) If T_1 and T_2 are theories, then $T_1 \cap T_2$ is a theory.

c) If T is a satisfiable theory, then $\{\neg F \mid F \in T\}$ is a satisfiable theory.

d) If T is a complete theory, then $G \vee H \in T$ if and only if $G \in T$ or $H \in T$.

e) Let T be an arbitary theory. Then $\forall x(x = x) \in T$.

3. a) Show that quantifier elimination for linear arithmetic still works if we add the binary functions min and max.

b) Use the elimination to prove that the following formula holds:

$$\forall a \forall b \forall c \forall d (\max(a, b) < \min(c, d) \rightarrow (\forall x(x > 2a \vee x \leq a) \vee \exists y(0 \leq y \wedge \max(c, y) = c))$$

c) Show that we can also add the function $|\cdot|$ that yields the absolute value of its argument.