Technische Universität München
I7
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Logic

Exercise Sheet 6

Discussion: June 5, 2014

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1. Apply predicate resolution to show the validity of the following formula:

$$F = \forall y Q(f(a), f(y)) \wedge \forall x \forall y (Q(y, f(y)) \rightarrow P(f(x), g(y, b)))$$
$$\rightarrow \ \exists x \exists y \exists z (P(x, y) \wedge P(f(a), g(x, b)) \wedge Q(x, z))$$

2. Recall the reduction from the halting problem for **goto**-programs to the validity of predicate logic and show that for any model $\mathcal{B}$ of $\psi_{P\beta}$ we have

$$<^{\mathcal{B}} \cap (D \times D) = \ \{(d_i, d_j) \mid i, j \in \mathbb{N}, i < j\}$$

where $d_i = (f^i(\mathbf{0}))^{\mathcal{B}}$ for each $i \geq 0$ and $D = \{d_i \mid i \geq 0\}$.

3. Recall the reduction from the tiling problem to satisfiability of predicate logic and show that the plain can be tiled with S and relations H and V if and only if $\phi_{S,H,V}$ is satisfiable.

4. Let $\mathcal{A}$ be a finite structure (i.e. $U_{\mathcal{A}}$ is finite) that is suitable for some finite signature S. Show that $Th(\mathcal{A})$ is finitely axiomatizable, i.e. axiomatizable by a finite set.

5. Show that the undecidability of $Th(\mathbb{N}, 0, 1, +, \cdot, =)$ implies the undecidability of $Th(\mathbb{Z}, 0, 1, +, \cdot, =)$. More concretely, for the signature $S = \{0, 1, +, \cdot, =\}$ provide a computable translation T from predicate logic formulas over S to predicate logic formulas over S such that for each such formula F we have that $F \in Th(\mathbb{N}, 0, 1, +, \cdot, =)$ if and only if $T(F) \in Th(\mathbb{Z}, 0, 1, +, \cdot, =)$.

Hints. Note that we do not have $<$ in our signature. You may use a result due to Lagrange.