

Logic

Exercise Sheet 4

Discussion: May 15, 2014

1. Give a satisfiable formula in predicate logic with/without equality such that each model has at least four elements.
2. Give a satisfiable formula in predicate logic with equality that only has finite models. Does such a formula in predicate logic without equality exist?
3. Give a satisfiable formula in predicate logic with/without equality that only has infinite models.
4. Prove that each formula containing only $\wedge, \vee, \forall, \exists, \rightarrow$ and atomic formulas is satisfiable.
5. Define a translation T such that for each formula F of predicate logic the following holds:
 - $\mathsf{T}(F)$ is a formula of predicate logic without function symbols and without equality.
 - F is satisfiable if and only if $\mathsf{T}(F)$ is satisfiable.
6. Give the Skolem normal form of the formulas
 - $F = \forall x \exists y \exists w (\neg P(a, w) \vee Q(f(x), y))$
 - $G = \forall y \neg ((P(b, g(x)) \vee \forall x Q(f(x))) \wedge R(y))$
7. Let the formula

$$F = \forall x \forall y \forall z P(x, f(y), g(z, x))$$

be given.

- Give a Herbrand structure for F that is not a model of F .
- Give a Herbrand structure for F that is a model of F .