Technische Universität München
I7
Stefan Göller ST 2014

Logic

Exercise Sheet 2

Discussion: April 24, 2014

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1. Prove that any formula built from atomic formulas and the operators $\wedge, \vee, \rightarrow$ is satisfiable.

2. Check the following formulas for satisfiability using one of the algorithms seen in the lecture:

- $F_1 = (\neg A \vee \neg D \vee B) \wedge D \wedge \neg B \wedge E \wedge (\neg D \vee \neg E \vee C)$
- $F_2 = (A \rightarrow C) \wedge (C \rightarrow E) \wedge (1 \rightarrow B) \wedge (C \wedge A \rightarrow D) \wedge (B \rightarrow A) \wedge$
  $(B \rightarrow E) \wedge (D \wedge E \rightarrow 0)$
- $F_3 = (\neg A \vee B) \wedge (\neg D \vee \neg E \vee C) \wedge C \wedge (\neg C \vee E \vee B) \wedge \neg B \wedge (C \rightarrow A)$
- $F_4 = (A \rightarrow E) \wedge (B \rightarrow 0) \wedge (C \rightarrow B) \wedge (1 \rightarrow A) \wedge (E \wedge B \rightarrow C) \wedge (C \rightarrow D)$

3. *Prove or disprove:* The resolution of a Horn formula F always yields a Horn formula, i.e. $\mathsf{Res}^*(F)$ is a Horn formula.

4. Prove that there is no Horn formula H with $H \equiv A \vee B$ by recapitulating the correctness proof of the marking algorithm.

5. Give a polynomial time algorithm that computes the following:

INPUT: A propositional formula F.

OUTPUT: A propositional formula G such that F is valid if and only if there exists a Horn formula H such that $H \equiv G$.

6. Give a polynomial time algorithm for the following problem (also known as 2SAT):

INPUT: A CNF formula $F = \bigwedge_{i=1}^{n} \bigvee_{j=1}^{m_i} L_{i,j}$, where $m_i \leq 2$ for each $i \in \{1, \ldots, n\}$.

QUESTION: Is F satisfiable?