Technische Universität München
I7
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Logic

Exercise Sheet 1

Discussion: April 10, 2014

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1. Assume F and G are two propositional formulas whose sets of occurring atomic formulas are disjoint. Prove the equivalence of the following two statements.

a) $F \models G$.

b) F is unsatisfiable or G is valid.

2. Prove the Substitution Theorem.

3. Use the semantical equivialences given in the slides (together with the Substitution Theorem) to transform the following formula F (resp. G) stepwise into CNF (resp. DNF). For every step state the used equivalence.

$$F = ((A \leftrightarrow \neg A) \vee (\neg C \wedge B)) \qquad G = \neg(A \rightarrow \neg(\neg B \vee \neg C))$$

4. Compute the truth table of $H = ((A \leftrightarrow B) \vee ((\neg C \wedge \neg B) \wedge A))$ and determine from it a semantically equivalent formula in CNF and DNF.

5. The *size* $|F|$ of a propositional formula F is inductively defined as follows: $|A_i| = 1$ for each $i \in \mathbb{N}$, $|(F \wedge G)| = |(F \vee G)| = |F| + |G| + 1$ and $|\neg F| = |F| + 1$.

Specify a family of "small" propositional formulas

$$\{F_n \mid n \geq 1, n \text{ is a power of two}\}$$

such that each formula F_n describes the parity function on n inputs, i.e. we have

- the atomic formulas of F_n are $A_1, \ldots, A_n$,
- for each suitable assignment $\mathcal{A}$ we have $\mathcal{A} \models F_n$ if and only if

$$|\{i \in \{1, \ldots, n\} \mid \mathcal{A}(A_i) = 1\}|$$

  is odd, and
- $|F_n| \in O(n^2)$.