Producing answers

Exercise: compute $2 + 2$ (challenging :-)))

We use the recursive definition of '+':

$$0 + n = n \quad \text{und} \quad (n + 1) + m = (n + m) + 1$$

We define a ternary predicate symbol $Sum(x, y, z)$ with intended meaning: 'the sum of x and y is equal to z'.

$$\begin{aligned} F_1 &= \forall x\ Sum(0, x, x) \\ F_2 &= \forall x \forall y \forall z\ (Sum(x, y, z) \rightarrow Sum(s(x), y, s(z))) \end{aligned}$$

Does $\{F_1, F_2\} \models F$ hold for $F = \exists x\ Sum(s^2(0), s^2(0), x)$?

$\{F_1, F_2\} \models F$ iff $F_1 \wedge F_2 \wedge \neg F$ unsatisfiable.

Clause form of $F_1 \wedge F_2 \wedge \neg F$:

$$\{ \ \{Sum(0, x, x)\}, \{\neg Sum(x, y, z), Sum(s(x), y, s(z))\}, \{\neg Sum(s^2(0), s^2(0), x) \ \}$$

Since the empty clause is derivable we have $\{F_1, F_2\} \models F$.
The 'value' of x such that $sum(s^2(0), s^2(0), x)$ can be read out from the substitutions.

Towers of Hanoi

In the great temple at Benares beneath the dome that marks the centre of the world, rests a brass plate in which are fixed three diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed sixty-four discs of pure gold, the largest disk resting on the brass plate, and the others getting smaller and smaller up to the top one. This is the tower of Bramah. Day and night unceasingly the priest transfer the discs from one diamond needle to another according to the fixed and immutable laws of Bramah, which require that the priest on duty must not move more than one disc at a time and that he must place this disc on a needle so that there is no smaller disc below it. When the sixty-four discs shall have been thus transferred from the needle which at creation God placed them, to one of the other needles, tower, temple, and Brahmins alike will crumble into dust and with a thunderclap the world will vanish.

Exercise: compute a sequence of moves that transport n discs from the first to the second peg.

Idea: Trivial for $n = 1$. For $n > 1$:

> move the upper $n - 1$ discs from the first to the third peg
>
> move the lowest dics to the second peg
>
> and move the $n - 1$ discs from the third to the second peg.

The formula

We define a predicate symbol $Move(n, x, y, z)$ of arity 4 with intended meaning: 'it is possible to move n discs from peg x to peg y usng peg z for temporary storage'

$$\begin{aligned} F_1 &= \forall x \forall y \forall z \; Move(0, x, y, z) \\ F_2 &= \forall n \forall x \forall y \forall z \quad (\,(Move(n, x, z, y) \wedge Move(n, z, y, x)) \\ &\qquad\qquad \longrightarrow \\ &\qquad\qquad Move(s(n), x, y, z)) \end{aligned}$$

Does $\{F_1, F_2\} \models Move(s^4(0), first, second, third)$ hold?

Horn clauses

- Fact clauses are positive one-literal clauses

$$\{ \; Sum(0, x, x) \; \} \; , \; \{ \; Move(0, x, y, z) \; \}$$

- Procedural clauses are of the form $\{P, \neg Q_1, \ldots, \neg Q_n\}$.
  P is the head and $Q_1, \ldots, Q_n$ the body.

$$\{ \; \neg Sum(x, y, z) \; , \; Sum(s(x), y, s(z)) \; \}$$
$$\{ \; \neg Move(n, x, z, y) \; , \; \neg Move(n, z, y, x)) \; , \; Move(s(n), x, y, z) \; \}$$

- Goal clauses are negative one-literal clauses

$$\{ \; Sum(s^2(0), s^2(0), x)\} \; , \; \{Move(s^4(0), erste, zweite, dritte) \; \}$$

All of them are predicate Horn clauses

Prolog

Prolog program = set of fact and procedure clauses.

The question to be answered is described by a goal clause.

The Prolog interpreter tries to solve the problem using SLD-Resolution.

The primitive data structures are terms, which allow to easily encode lists and trees.

Many deviations from a 'pure' solution using only predicate logic.