Lemma (one-literal rule)

If B_{i+1} is obtained from B_i by an application of the one-literal rule, then B_{i+1} contains a sat. formula $\boxed{\text{iff}}$ B_i contains a sat. formula.

Proof Let $B_i = F \cup \{F_1, \ldots, F_n\}$

$B_{i+1} = G \cup \{F_1, \ldots, F_n\}$

($\Rightarrow$) Assume some formula of B_i is sat.

We prove some formula of B_{i+1} is sat

If the formula is some F_i ✓

Assume all F_i are unsat and F is sat.

We prove that G is sat

Let A be assignment with $\boxed{A(F) = 1}$.

Since $F = F' \cup \{L\}$ we have $\boxed{A(L) = 1}$

We prove $A(G) = 1$

It suffices to show that $A(C) = 1$ for every clause C of G.

There are two cases

1) F contains the clause $C \cup \{\bar{L}\}$

Since $A(F) = 1$ we have $A(C \cup \{\bar{L}\}) = 1$

Since $A(L) = 1$ we get $A(C) = 1$

2) F contains the clause C

Then since $A(F) = 1$ we have $A(C) = 1$

$\Leftarrow$ Assume some formula of B_{i+1} is sat

We prove that some formula of B_i is sat

If the formula is some F_i ✓

Assume all F_i's are unsat and G is sat

We prove that F is sat

Let A be assignment with $A(G) = 1$

We show that the assignment A' that extends A with $A'(L) = 1$ satisfies $A'(F) = 1$

Let C be a clause of F. There are 3 cases

1) C contains neither L nor $\bar{L}$

Then C is clause of G and so $A(C) = 1$ which implies $A'(C) = 1$

2) C contains L

Then $A'(C) = 1$ because $A'(L) = 1$

3) C contains $\bar{L}$ but not L

Then $C = C' \cup \{\bar{L}\}$ and C' is clause of G. Since $A(G) = 1$ we have $A(C') = 1$ and so $A'(C) = 1$

$\boxtimes$