4				9				2
		1				5		
	9		3	4	5		1	
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Sudoku

4				9				2
		1				5		
	9		3	4	5		1	
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Fill the empty squares so that every row, column, and 3x3 block contains all digits from 1 to 9.

Why 6?

4				9				2
		1				5		
	9		3	4	5		1	6
		8				2	5	
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	
		9				6		
5				7				4

Why 6?

4				9				2
		1				5		
	9		3	4	5		1	6 7
		8				2	5	3 7
7		5		3		4	6	1
	4	6				9		8
	6		1	5	9		8	3 7
		9				6		
5				7				4

A Very Brief History of Logic

Origin in Ancient Greece: Aristotle (384–322 BC) studies the nature of arguments and deduction.

Several works, e.g. *Analytica priora*, *Analytica posteriora*.

Systematic study of syllogisms.

Syllogisms

A syllogism is discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last phrase that they produce the consequence, and by this, that no further term is required from without in order to make the consequence necessary.

Aristotle, *Analytica Priora*

Syllogisms

If all men are mortal and
Socrates is a man,
then Socrates is mortal.

If a number is even and bigger than two,
then it is not a prime.

If interest rates are high,
then stockbrokers are unhappy.

Syllogisms: reinterpreting Aristotle

A syllogism is discourse in which, certain things being stated, and even assuming that we only know the meaning of "if, then, and, is (are), all, some, none", something other than what is stated follows of necessity from their being so.

If all X are Y and
S is X,
then S is Y.

If X is Y and Z,
then X is not P.

If X are Y,
then Z are W.

Syllogisms

Aristotle compiled a list of the valid syllogisms.

All spaniels are dogs	All P are M	
All dogs are animals	All M are S	(Barbara)
All spaniels are animals	All P are S	

No flower is an animal	No P is M	
Alle dogs are animals	All S are M	(Cesare)
No flower is a dog	No P is S	

All dolphins live in the see	All M are P	
All dolphins are mammals	All M are S	(Darapti)
Some mammals live in the see	Some S are P	

Criticism (with hindsight)

Many valid deductions are not contained in Aristotle's list, e.g.

$$\frac{\text{Some cat is feared by every mouse}}{\text{All mice fear at least one cat}} \qquad \frac{\text{Some A are B}}{\text{All C are D}}$$

Aristotle does not provide any calculus to handle long chains of deductions.
(About 2000 years later, Leibniz was the first to envision such a calculus.)

Propositional logic

George Boole (1815 – 1864)

Atomic propositions can be true or false (we do not know which!)
Linked using operators (and, or, not, if-then . . .).
No quantification (all, some).

Example:

- Propositions: “Alice is an architect”, “Bob is a lawyer”.
- Four possible situations or worlds:
  (1) Alice is an architect, Bob is a lawyer.
  (2) Alice is an architect, Bob is not a lawyer.
  (3) Alice is not an architect, Bob is a lawyer.
  (4) Alice is not an architect, Bob is not a lawyer.

- Some of the possible combined propositions:
  "Alice is an architect and Bob is a lawyer".
  "If Alice is an architect then Bob is a lawyer".
  "If Alice is not an architect, then Bob is not a lawyer".
  "If Bob is not a lawyer, then Anne is not an architect".

B is a consequence of A: B is true in all worlds in which A is true.

Algebraic calculus to decide whether B is a consequence of A.
The calculus is based on the analogy between true and 1, false and 0,
or and addition, and and multiplication.

Pradicate logic

Frege, Peano, Russell (end of the 19th century)

Logic as foundation of mathematics, as formal tool for avoiding contradictions.

Development of predicate logic, which allows to

- describe relations between entities.
- formulate existential statements: "there is an x such that ... holds".
- formulate universal statements: "for all x ... holds".

If there exists a cat that is feared by every mouse then
for every mouse there exists a cat that the mouse fears

Logic in Computer Science

Shannon (1916 – 2001) shows in 1937 that propositional logic (boolean algebra) can be used to describe and optimize electromechanical circuits.

Newell, Simon, Robinson develop in 1950-1960 the first systems for the mechanization of logical deduction, and apply them to artificial intelligence problems.

Applications in computer science (I)

- Digital circuit design: digital circuits canbe described as logical formulas $\rightsquigarrow$ Circuit design and optimization
- Models and specifications: Precise description of comlex systems and requirements
- Verification: proving that a program satisfies its specification
- Databases: Formalization of queries
  $\rightsquigarrow$ Query language SQL (Structured query language)

Applications in computer science (II)

- Artificial Intelligence:
  - Planning
  - Human-Computer Interaction
  - Theorem provers: automated or computer assisted mathematical proofs $\rightsquigarrow$ automatic of important theorems in boolean algebras, four-color theorem, Kepler's conjecture . . .
- Logic programming: PROLOG

Quoting Edsger W. Dijkstra:

> Informatics = VLSAL (Very Large Scale Application of Logics)

Quoting Georg Gottlob:

> Computer science is the continuation of logic by other means

Problems of natural languages (I)

Problem: Assigning truth values to natural language sentences is difficult.

Examples:

- I have only drunk a little.
- His money went up in smoke.
- Revenge is a dish best served cold.

Problems of natural languages (II)

Problem: Natural language can be difficult to understand when precision is required.

Example: Quote from *Analytica Priora* by Aristotle

The statement If the middle term is related universally to one of the extremes, a particular negative syllogism must result whenever the middle term is related universally to the major whether positively or negatively, and particularly to the minor and in a manner opposite to that of the universal statement: by 'an opposite manner' I mean, if the universal statement is negative, the particular is affirmative: if the universal is affirmative, the particular is negative.

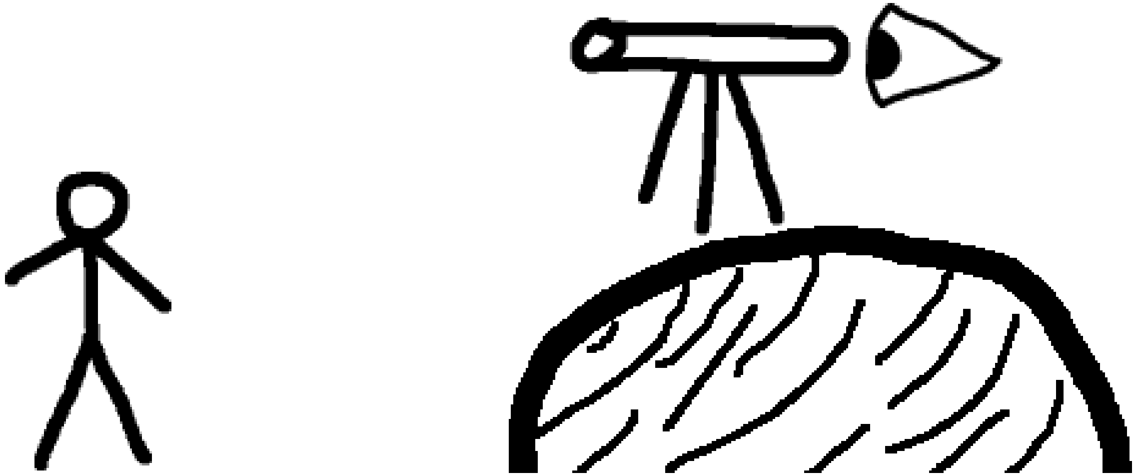
The proof For since the negative statement is convertible, N will belong to no M: but M was admitted to belong to some O: therefore N will not belong to some O: for the result is reached by means of the first figure. Again if M belongs to all N, but not to some O, it is necessary that N does not belong to some O: for if N belongs to all O, and M is predicated also of all N, M must belong to all O: but we assumed that M does not belong to some O. And if M belongs to all N but not to all O, we shall conclude that N does not belong to all O: the proof is the same as the above. But if M is predicated of all O, but not of all N, there will be no syllogism.

Problems of natural language (III)

Problem: Natural languages are ambiguous.

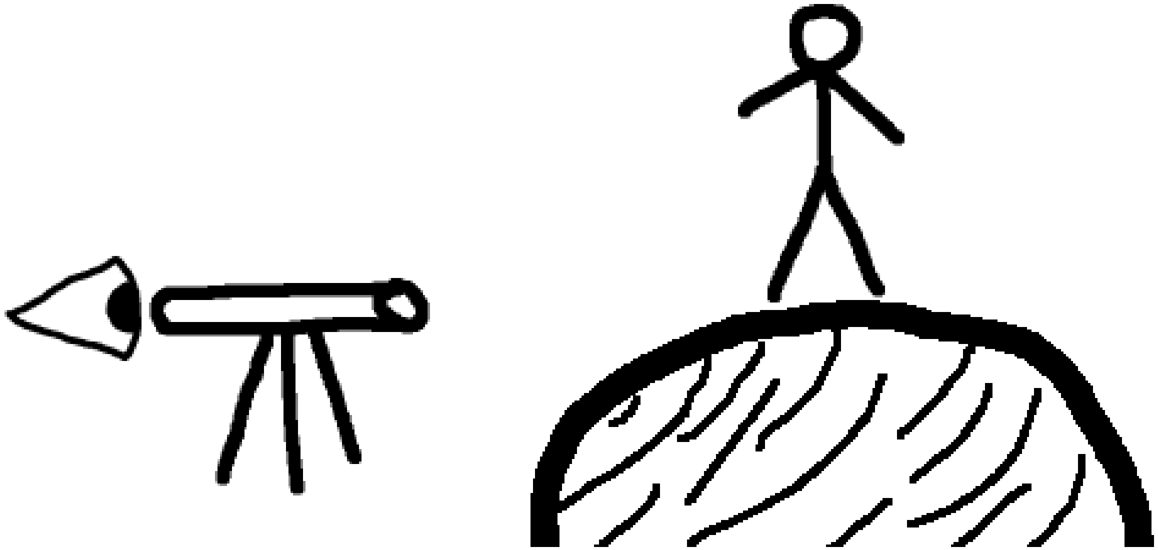
Example: I saw the man on the hill with the telescope.

I saw the man . . .



(((I saw the man) on the hill) with the telescope)

I saw the man . . .

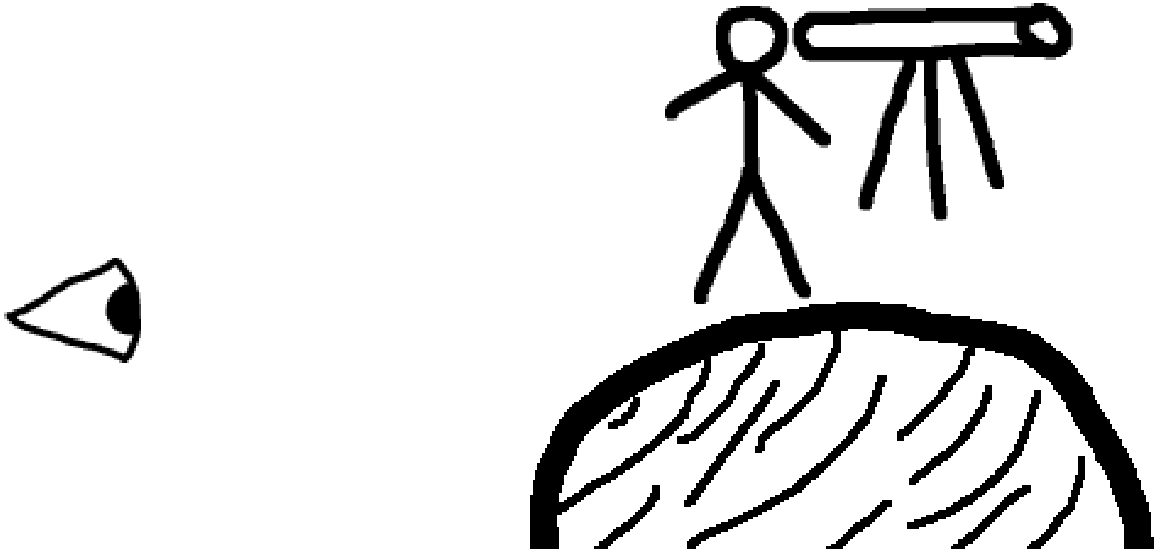


((I saw (the man on the hill)) with the telescope)

I saw the man . . .

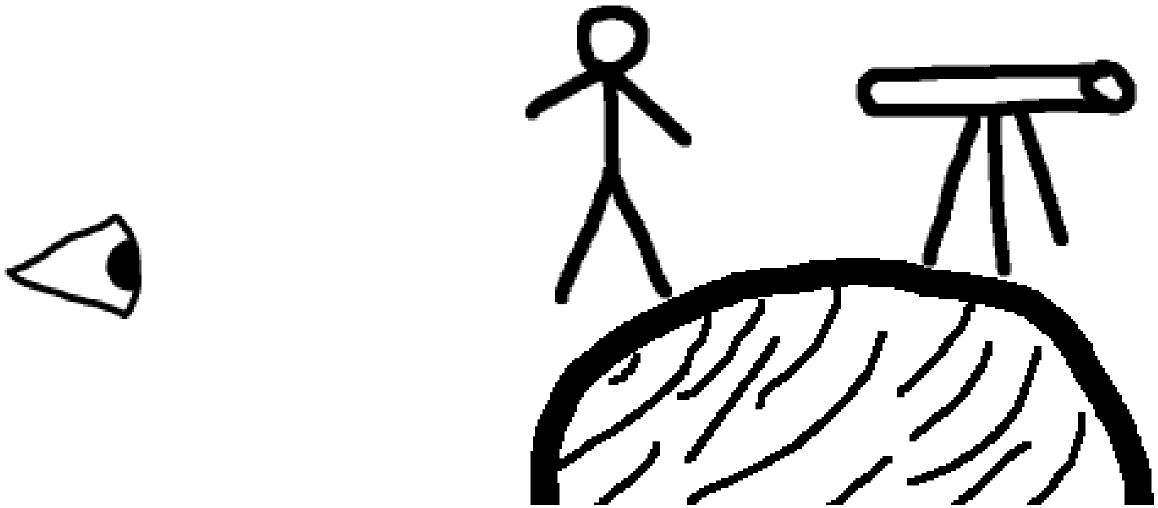
((I saw the man) (on the hill with the telescope))

I saw the man . . .



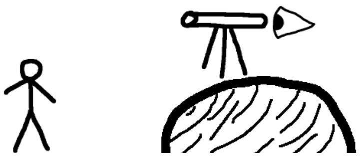
(I saw ((the man on the hill) with the telescope))

I saw the man . . .

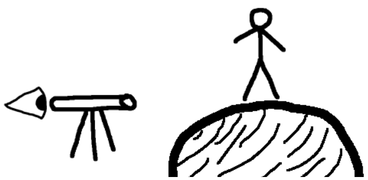


(I saw (the man (on the hill with the telescope)))

I saw the man . . .

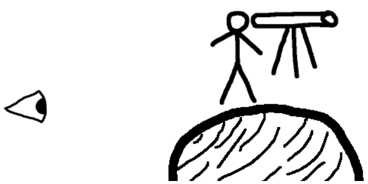


(((I saw the man) on the hill) with the telescope)

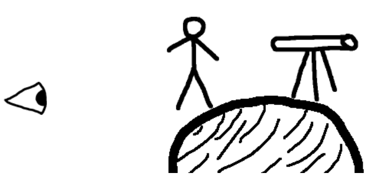


((I saw (the man on the hill)) with the telescope)

((I saw the man) (on the hill with the telescope))



(I saw ((the man on the hill) with the telescope))



(I saw (the man (on the hill with the telescope)))

5 possible interpretations

Problems of natural languages (IV)

Problem: Natural languages are not "context-free".

The Beatles are musicians
Paul McCartney is a Beatle

Paul McCartney is a musician

The Beatles are four
Paul McCartney is a Beatle

Paul McCartney is four