Logic – Homework 7

Discussed on 04.07.2011.

Exercise 7.1 Unification Warm-Up

Which of the following unification problems are solvable? Give the most general unifier if it exists.

(a) $f(x, y) \stackrel{?}{=} f(h(a), x)$

(b) $f(x, y) \stackrel{?}{=} f(h(x), x)$

(c) $f(x, b) \stackrel{?}{=} f(h(y), z)$

(d) $f(x, x) \stackrel{?}{=} f(h(y), y)$

Exercise 7.2 Unification w/o occur check

Several implementations of unification algorithms (e.g. the one used in Prolog) omit the occur check for efficiency reasons. This means, they do not check whether the variable on the one side occurs in the term on the other side.

Give an example of a non-unifiable set of literals with two elements, so that such a unification algorithm would, depending on the implementation, run into an inifite loop, or wrongly yield "unifiable". The two literals should not have any variables in common.

Exercise 7.3 First-Order-Resolution

Given is the following formula already presented in exercise 6.4:

$$\forall y.Q(f(a), f(y)) \wedge \forall xy.(Q(y, f(y)) \rightarrow P(f(x), g(y, b))) \rightarrow \exists xyz.(P(x, y) \wedge P(f(a), g(x, b)) \wedge Q(x, z))$$

This time use the first-order resolution presented in the lecture to show its validity.

Exercise 7.4 Modelling

(a) Show that a relation that is total, transitive and symmetric is also reflexive. To this end, give a formula describing this theorem and use first-order resolution to prove its validity.

(b) In first-order logic **without** equality, model the properties of the predicate $sum(x, y, z)$ together with the successor-function $succ(x)$ and a constant 0, so that they reflect the well-known behavior of sum, e.g. $sum(x, y, z) \rightarrow sum(y, x, z)$. Then use this to prove $2 + 2 = 4$ by resolution.