Note Title 04.07.2011

Exercise 7.1 **Unification Warm-Up**

Which of the following unification problems are solvable? Give the most general unifier if it exists.

(a) $f(x,y) \stackrel{?}{=} f(h(a),x)$ $sub = \{x \mapsto h(a),\ y \mapsto h(a)\}$

(b) $f(x,y) \stackrel{?}{=} f(h(x),x)$ $sub(x) \neq sub(h(x))$

(c) $f(x,b) \stackrel{?}{=} f(h(y),z)$ $sub = \{x \mapsto h(y),\ z \mapsto b\}$

(d) $f(x,x) \stackrel{?}{=} f(h(y),y)$ $h(y) \neq y \Rightarrow$ no subst for x

Exercise 7.2 **Unification w/o occur check**

Several implementations of unification algorithms (e.g. the one used in Prolog) omit the occur check for efficiency reasons. This means, they do not check whether the variable on the one side occurs in the term on the other side.

Give an example of a non-unifiable set of literals with two elements, so that such a unification algorithm would, depending on the implementation, run into an inifite loop, or wrongly yield "unifiable". The two literals should not have any variables in common.

$$f(x, x) \stackrel{?}{=} f(h(y), y)$$

7.3

$\forall y.Q(f(a), f(y)) \wedge \forall xy.(Q(y, f(y)) \rightarrow P(f(x), g(y, b))) \rightarrow \exists xyz.(P(x, y) \wedge P(f(a), g(x, b)) \wedge Q(x, z))$

7. 4.

(a) Show that a relation that is total, transitive and symmetric is also reflexive. To this end, give a formula describing this theorem and use first-order resolution to prove its validity.

total : $\forall x.\ \exists y.\ P(x,y)$ I

transitive: $\forall xyz.\ (P(x,y) \wedge P(y,z) \rightarrow P(x,z))$ II

symm.: $\forall xy.\ (P(x,y) \rightarrow P(y,x))$ III

to show : reflexive: $\forall x.\ P(x,x)$ IV

$\mathrm{I} \wedge \mathrm{II} \wedge \mathrm{III} \rightarrow \mathrm{IV}$ | $\mathrm{I} \wedge \mathrm{II} \wedge \mathrm{III} \wedge \neg \mathrm{IV}$ unsat

Remember: $F \rightarrow G$ is valid

show $\neg(F \rightarrow G)$ is unsat

$F \wedge \neg G$ is unsat

$$\forall x_1 \exists y_1 \, P(x_1, y_1)$$
$$\land \; \forall x_2 \forall y_2 \, (\neg P(x_2, y_2) \lor P(y_2, x_2))$$
$$\land \; \forall x_3 y_3 z_3 \, (\neg P(x_3, y_3) \lor \neg P(y_3, z_3) \lor P(x_3, z_3))$$
$$\land \; \exists x_4 \neg P(x_4, x_4)$$

$$\equiv_s \; \exists x_4 \forall x_1 \exists y_1 \forall x_2 y_2 x_3 y_3 z_3 \quad F$$

$$\forall \ldots \; P(x_1, f(x_1)) \quad (1)$$
$$\land \; (\neg P(x_2, y_2) \lor P(y_2, x_2)) \quad (2)$$
$$\land \; (\neg P(x_3, y_3) \lor \neg P(y_3, z_3) \lor P(x_3, z_3)) \quad (3)$$
$$\land \; \underline{\neg P(a, a)} \quad (4)$$

$\{P(x_1, f(x_1))\}$ $\qquad$ $\{\neg P(x_2, y_2) \vee P(y_2, x_2)\}$

$\{y_2 \mapsto f(x_1),\ x_2 \mapsto x_1\}$

$\{P(f(x_1), x_1)\}$

$\{\neg P(x_3, y_3), \neg P(y_3, z_3),$
$P(x_3, z_3)\}$

$y_3 \mapsto f(x_1)$
$z_3 \mapsto x_1 \quad x_3 \mapsto x_1$

$\{\neg P(x_1, f(x_1)), P(x_1, x_1)\}$ $\qquad$ $\{P(x_1, f(x_1))\}$

$\emptyset$

$\{\neg P(a, a)\}$ $\qquad$ $\{P(x_1, x_1)\}$

$x_1 \mapsto a$

$\square$

(b) In first-order logic **without** equality, model the properties of the predicate $sum(x, y, z)$ together with the successor-function $succ(x)$ and a constant 0, so that they reflect the well-known behavior of sum, e.g. $sum(x, y, z) \rightarrow sum(y, x, z)$. Then use this to prove $2 + 2 = 4$ by resolution.

Model sum() : $\forall xyz.\ sum(x, y, z) \rightarrow sum(y, x, z)$

$\forall xyz.\ sum(x, y, z) \rightarrow sum(succ(x), y, succ(z))$

$\forall x.\ sum(0, x, x)$

$\neg(sum(succ(succ(0)), succ(succ(0)), succ^4(0))$

Clauses : $\{\neg sum(x_1, y_1, z_1), sum(y_1, x_1, z_1)\}$

$\{\neg sum(x_2, y_2, z_2), sum(succ(x_2), y_2, succ(z_2))\}$

$\{sum(0, x_3, x_3)\}$

$\{\neg sum(succ^2(0), succ^2(0), succ^4(0)\}$

$x_2 \mapsto succ(0),\ y_2 \mapsto succ^2(0),\ z_2 \mapsto succ^3(0)$

$\{\neg sum(succ(0), succ^2(0), succ^3(0))\}$

$x_2 \mapsto 0,\ y_2 \mapsto succ^2(0),\ z_2 \mapsto succ^2(0)$

$\{\neg sum(0, succ^2(0), succ^2(0))\}$

$x_3 \mapsto succ^2(0)$

$\square$