Logic – Homework 3

Discussed on 30.05.2011.

Exercise 3.1

Let $F = (x \vee y) \rightarrow z$ and $G = x \leftrightarrow (y \wedge z)$, where $x < y < z$.

(a) Draw the BDDs of F and G.

(b) During the lecture an algorithm implementing the OR-operation on BDDs has been presented. Use this algorithm to construct a BDD for $F \vee G$.

(c) Modify this algorithm to receive the BDD for $F \wedge G$.

(d) Let $H = ((z \leftrightarrow (x \oplus y)) \wedge (w \leftrightarrow (y \vee z)) \vee x \vee (u \leftrightarrow (x \vee w))$. Draw the BDD of H. How many satisfiable assignments exist for H? How could one read this directly from the BDD?
Hint: For each node give the number of satisfying assignments for the subtree starting at that node. Start with the "lowest" one.
Hint #2: You may either calculate the BDD yourself or may use one of the tools presented on the homepage.

Exercise 3.2

There exists a variant of BDDs that quite often produce smaller graphs: *BDDs with complement edges* (CBDDs). An example for such a CBDD can be found further down. The main differences between a normal BDD and a CBDD are as follows:

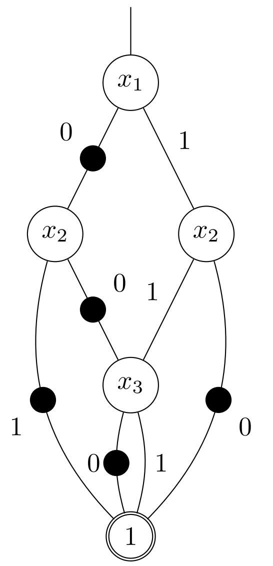
- In a CBDD, there are two kinds of edges:

  Postive (normal) edges, drawn as usual; and negative (complement) edges, explicitly marked by a black dot.
- The root of the CBDD has an incoming edge (which can be negative).
- In a CBDD there is no node labeled by 0.

A negative edge tells us to take the complement of the formula represented by the node the edge points to:

For instance, consider the node labeled by x_3 in the CBDD depicted below: its two children both represent the formula $\top$, but as the edge to its 0-child is negative, the node represents the formula $\neg x_3 \wedge \neg\top \vee x_3 \wedge \top \equiv x_3$.

For another example, consider the left of the two nodes labeled by x_2: this node represents the formula $\neg x_2 \wedge \neg x_3 \vee x_2 \wedge \neg\top \equiv \neg x_2 \wedge \neg x_3$.



(a) State the formula represented by the CBDD above. Also give its truth table.

(b) CBDDs in general are not unique, i.e. for one formula there might be multiple CBDDs representing that formula. Find two formulas that can be represented by at least two CBDDs each. Also state these CBDDs.

(c) If one omits negative 0-edges, it is possible to create unique CBDDs. Therefore show, that for a function $ite(x, F, G) \equiv x \wedge F \vee \neg x \wedge G$ (*if then else*) the following equivalence holds:

$$ite(X, F, \neg G) \equiv \neg ite(x, \neg F, G)$$

Use this equivalence to transform the example CBDD from above into a unique one, which does not contain negative 0-edges.