$\forall xs \in \mathcal{L}(X)\ \underbrace{\forall ys \in \mathcal{L}(X)\ \forall zs \in \mathcal{L}(X):}_{H(xs)}$ IH.

$$(xs @ ys) @ zs = xs @ (ys @ zs)$$

IH: $\forall xs \in \mathcal{L}(X) : H(xs)$

Base: $H(nil)$?

$\forall ys\ \forall zs : (nil @ ys) @ zs = nil @ (ys (@ zs))$

Sei: ys und zs beliebig.

$\underbrace{(nil @ ys)}_{\text{def } @} @ zs = (ys) @ zs$ def @ $\xrightarrow{1}$

$= ys @ zs$ def parenth.

$= (ys @ zs)$ def parenth

$= nil @ (ys @ zs)$ def@ $\xleftarrow{1}$

Hence $H(nil)$

$= \cancel{xs @}$

Step: $\cancel{H(xr) \Rightarrow}$ ~~Sei~~ Sei: x, xr, xs beliebig sodaß $xs = x :: xr$.

$H(xr) \to H(xs)$

$(\forall ys\ \forall zs : (xr @ (ys @) zs) = xr @ (ys) @ zs))$

$\to (\forall ys\ \forall zs : (x :: xr @ ys) @ zs = x :: xr @ (ys @ zs)).$

Sei ys und zs beliebig und ~~setze~~ assume $H(xr)$.

Zu zeigen:

$(x :: xr @ ys) @ zs = x :: xr @ (ys @ zs)$

$\underbrace{(x :: xr @ ys)}_{\text{def } @} @ zs = (x :: (xr @ ys)) @ zs$ def @ 2 $\rightarrow$

$= \cancel{xs @}$

$x :: ((xr @ ys) @ zs)$ def @ 2 $\rightarrow$

$= x :: (xr @ (ys @ zs))$ $H(xr)$

$= (x :: xr) @ (ys @ zs)$ def @ 2 $\leftarrow$

$= xs @ (ys @ zs)$

Hence $H(xs)$