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Fundamental Algorithms

Exercise Sheet 1

1. Consider the following description of the multiplication *à la russe*:

   > Write the multiplicand and the multiplier, each at the head of a column. Repeat the following rule until the number in the left-hand column is 1: divide the number in the left-hand column by 2, ignoring any fractions, and double the number in the right-hand column. Next, cross out each row in which its left-hand side is even. The multiplication result is obtained by adding the remaining numbers in the right-hand column.

   The following illustrates how to multiply 29 with 52:

29	52
~~14~~	~~104~~
7	208
3	416
1	832
	1508

   Prove that for any positive integers m and n, multiplication *à la russe* always returns $m \cdot n$.

2. Write down the algorithm multiplication *à la russe* using the notation introduced in the lecture. The algorithm should take two positive integers m and n, and compute the product in the variable p. Prove the correctness of your algorithm by using a loop invariant.

3. In Landau notation, we usually abuse notation by writing $f(n) = \mathcal{O}(g(n))$ to mean $f \in \mathcal{O}(g)$, where f and g are functions from natural numbers to real numbers. This convention is useful, since it allows us to write for instance $\mathcal{O}(n^2)$ without having to introduce a function for the polynomial n^2.

   With abuses of notation, determine whether the following statements are true or false. Justify your answers.

   (a) $10n + 40 = \mathcal{O}(n^2 + 2n + 1)$.

   (b) $\sqrt{n + 23} = \Theta(\sqrt{n})$.

   (c) $n^{1000} = \Omega(2^n)$.

   (d) $f(n) = o(g(n))$ if and only if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.

   (e) If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = L$, where $0 < L < \infty$, then $f(n) = \Theta(g(n))$.