Technische Universität München
Theoretische Informatik
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Winter term 2010

Cryptography – Mock Exam

Last name: ______________________

First name: ______________________

Student ID no.: ______________________

Signature: ______________________

Code $\in \{A, \ldots, Z\}^6$: ☐ ☐ ☐ ☐ ☐ ☐

- If you feel ill, let us know immediately.
- Please, **do not write** until told so. You are given approx. 10 minutes to read the exercises and address us in case of questions or problems.
- You will be given **90 minutes** to fill in all the required information and write down your solutions.
- Only fill in a **code** if you agree that your results are published under this code on a webpage.
- Don't forget to **sign**.
- Write with a non-erasable **pen**, do not use red or green color.
- You are not allowed to use **auxiliary means** other than your pen and a simple calculator.
- You may answer in **English or German**.
- Please turn off your **cell phone**.
- Check that you have received **9 sheets of paper** and, please, try to **not destroy the binding**.
- Write your **solutions** directly into the exam booklet.
- Should you require additional **scrap paper**, please tell us.
- You can obtain **40 points** in the exam. You need **17 points** in total to pass including potential bonuses awarded.
- See the next page for a list of **abbreviations**.
- Don't fill in the table below.
- Good luck!

Ex1	Ex2	Ex3	Ex4	Ex5	Ex6	Ex7	$\sum$

Exercise 1 **Yes/No** **each 1P=6P**

Points are rewarded as follows:

- Correct answer: 1P
- Incorrect answer: -1P
- No answer: 0P

The final number of points is the total if positive, otherwise zero.

Remark: See the last page for a list of abbreviations.

	true	false
If PRGs exist, then also PRFs exist.	☒	☐
From every OWF a PRG can be constructed.	☒	☐
You have seen in the lecture how to construct a family of CRHFs based on any OWF.	☐	☒
(*) Computational secret ES exist if and only if CCA-secure ES exist.	☒	☐
Existence of TDPs implies existence of CCA-secure PKES.	☒	☐
Existence of secure DSS is equivalent to the existence of CPA-secure ES.	☒	☐

(*) with $|\mathcal{K}| < |\mathcal{M}|$.

Exercise 2 **"One-liners"** **5P**

Give a short (one line) answer/explanation using the results from the lecture and the exercises.

(1P): Describe how a strong PRP can be constructed from a PRF F. (Assume F has key and block length n.)

Answer: 4-round Feistel network with 4 indep. keys

(1P): Show how to solve the DDH relative to $\mathsf{Gen}\mathcal{G}_{\mathbb{P}}$ in PPT. (Recall that Gen returns $I = (\langle \mathbb{Z}_p^*, 1, \cdot \rangle, q, g, x, h)$ with p a n-bit prime, $q = p - 1$, and $\langle g \rangle = \mathbb{Z}_p^*$.)

Answer: Compute $\left(\frac{g_3}{p}\right)$

(1P): Describe one construction which tries to fix the short key length of DES and is conjectured to be secure.

Answer: Triple DES: $DES_{k_1} \circ DES_{k_2}^{-1} \circ DES_{k_3}$

(1P): State the design principle on which AES and the DES-mangler function are based on.

Answer: Substitution-permutation network

(1P): State why the basic version of the RSA PKES should be used together with randomized padding, and name one padding conjectured to yield a CCA-secure PKES.

Answer: Otherwise it is not CPA secure; OAEP

Exercise 3

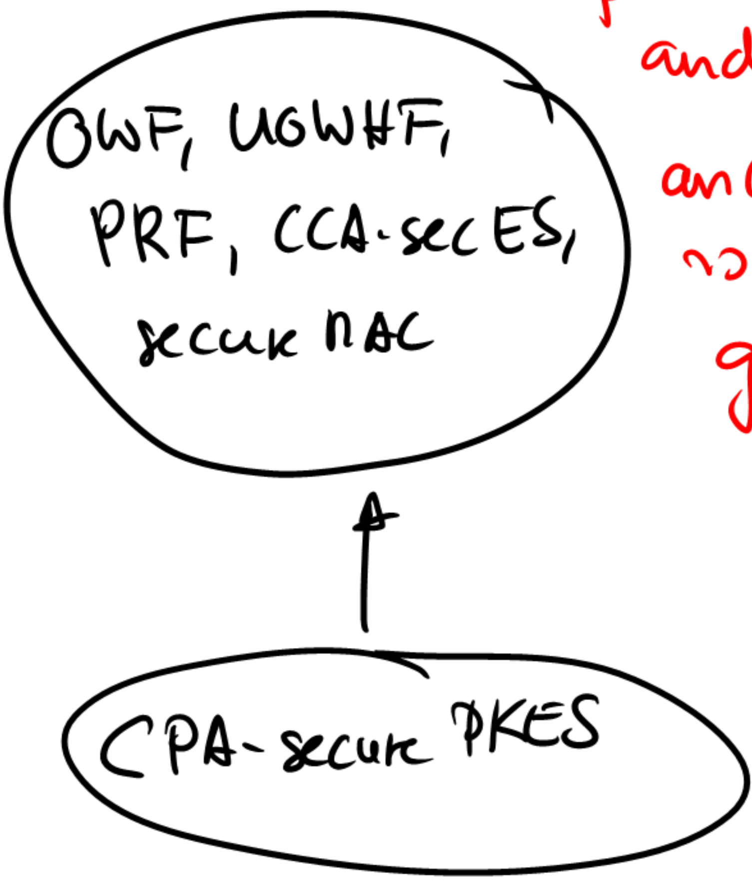
Draw a graph with nodes

$$\{\text{OWF}, \text{UOWHF}, \text{PRF}, \text{CCA-secure ES}, \text{secure MAC}, \text{CPA-secure PKES}\}$$

with an edge from node A to node B if the existence of A is *known* to imply the existence of B.

path

all in here can be constructed from an OWF, and also yields an OWF ⇝ complete graph



Exercise 4

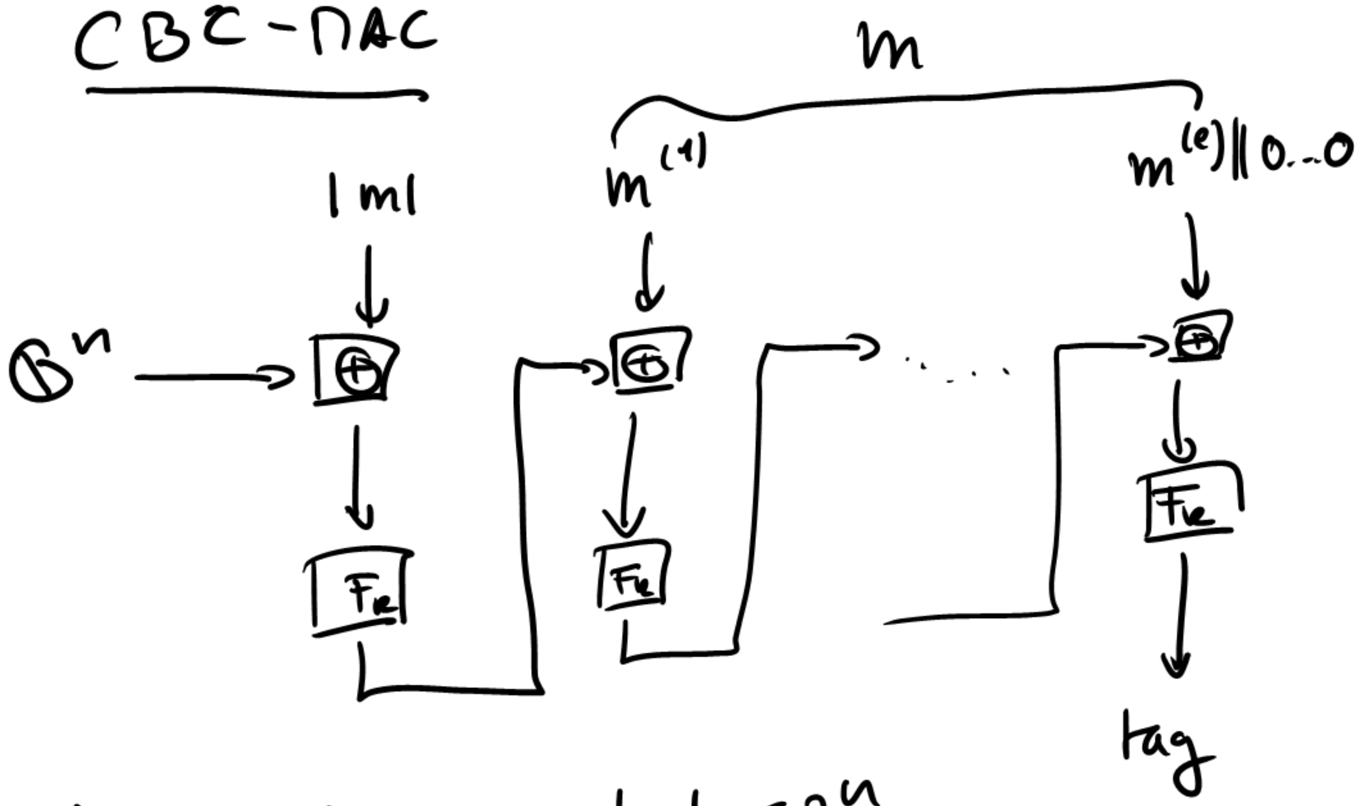
Let F be a PRF of key and block length n.

(a) Construct from F a secure MAC scheme for (almost) unrestricted message length. It suffices to define Mac and the padding function.

(b) Briefly describe how a CPA-secure ES and a secure MAC can always be combined into a CCA-secure ES.

Remark: There are several ways to solve (a). It suffice to give a single construction which can handle messages of length $< 2^n$. Don't forget to pad the actual message.

(a) CBC-MAC



Assumptions:

- $|m| < 2^n$ so that it can be encoded as an n-bit string
- $m^{(i)}$ is an n-bit block, the last block $m^{(\ell)}$ padded with zeros.

(b) <u>Enc-then-Mac:</u>

- Generate secret keys k_E for the ES and k_M for the MAC.

- For encryption:

  ① $c := \mathrm{Enc}_{k_E}(m)$

  ② $t := \mathrm{Mac}_{k_M}(c)$

  $\rightsquigarrow$ output $c \| t$

- For decryption:

  ① Check $\mathrm{Vrf}_{k_M}(m, t) = 1$

  - If not, output $\perp$

  ② $m := \mathrm{Dec}_{k_E}(c)$

  $\rightsquigarrow$ output m.

Exercise 5

Let F be a PRP of key and block length n. Define $T_k[t](x) := F_t(x \oplus F_k(t))$ for $t \in \{0,1\}^n$.

Show that T is not a secure TBC.

Reminder: Recall T is secure if PPT-Eve can only distinguish with negligible advantage between the following two oracles:

- $\mathcal{O}_T$: initializes itself by choosing $k \overset{u}{\in} \{0,1\}^n$; then answers a query (t, x) by $T_k[t](x)$.
- $\mathcal{O}_{\text{ideal}}$: has an independent instance $\mathcal{O}^t_{\text{perm}}$ of the random permutation oracle for every tweak $t \in \{0,1\}^n$, and answers a query (t, x) by $\mathcal{O}^t_{\text{perm}}(x)$.

• Recall <u>Recall</u> t is public, i.e. chosen by the adversary

$\rightsquigarrow$ ① $\sigma(\underset{= t}{0^n}, \underset{= x}{0^n}) =: y$

② $\underset{\text{PRP}}{F}{}^{-1}_{\underset{= t}{0^n}}(y) =: z$

③ $\sigma(0^n, z) =: w$

④ if $w = F_{0^n}(0^n)$ output TBC
else ideal

<u>If $\sigma = T$:</u>

$y = F_{0^n}(0^n \oplus F_k(0^n)) = F_{0^n}(F_k(0^n))$

$z = F_k(0^n)$

$w = F_{0^n}(F_k(0^n) \oplus F_k(0)^n) = F_{0^n}(0^n)$

<u>otherwise:</u> Prob. that $\sigma(0^n, z) = F_{0^n}(0^n)$ is negl

Exercise 6

Let $\mathbb{G} = \langle \mathbb{Z}_{23}^*, \cdot, 1 \rangle$.

(a) Show that $g = 5$ is a generator of $\mathbb{G}$.

(b) Compute all values of a run of the Diffie-Helman protocol for Bob's resp. Alice's secret exponent $b = 4$ resp. $a = 9$ and the shared group $\mathbb{G} = \mathbb{Z}_{23}^*$ with $g = 5$.

(c) Briefly describe how the DH protocol and the El Gamal PKES are related to each other.

(d) Let $\mathsf{Gen}\mathcal{G}$ be the DLP-generator used in an El Gamal PKES.

- Formally state the problem which needs to hard relative to $\mathsf{Gen}\mathcal{G}$ in order for the PKES, and describe such a conjectured generator.
- Propose a subgroup of $\mathbb{Z}_{23}^*$ which is better suited for the DH protocol and El Gamal.

  It suffices to state a generator and the size of the subgroup.

(a) $23 - 1 = 2 \cdot 11$

$\rightsquigarrow$ need to check that $5^2 \equiv 2 \not\equiv 1$ (✓)

and $5^{11} \equiv 25^5 \cdot 5$

$\equiv 2^5 \cdot 5$

$\equiv 32 \cdot 5$

$\equiv 9 \cdot 5 \equiv 45 \equiv 22 \equiv -1 \not\equiv 1$ (✓)

(b) $h_B = g^b \equiv 5^4 \equiv 2^2 \equiv 4 \ (23)$

$h_A = g^a \equiv 5^9 \equiv 4^2 \cdot 5 \equiv 80 \equiv 11 \ (23)$

$k = g^{ab} \equiv 4^9 \equiv 16^4 \cdot 4$

$\equiv (-7)^4 \cdot 4$

$\equiv 49^2 \cdot 4 \equiv 3^2 \cdot 4$

$\equiv 36 \equiv 13 \ (23)$

(c) El Gamal uses k as OTP for encrypting a "message group element" & a is chosen unif. at random for every encryption.

(d) The DDH needs to be hard wrt the used groups

$\leadsto$ ① $x \overset{u}{\in} \mathbb{Z}_q$, $g_1 := g^x$

② $y \overset{u}{\in} \mathbb{Z}_q$, $g_2 := g^y$

③ $b \overset{u}{\in} \{0,1\}$

④ if $b = 0$ then $g_3 \overset{u}{\in} \mathbb{G}$

else $g_3 := g^z$ for $z \overset{u}{\in} \mathbb{Z}_q$

⑤ $r := \mathcal{D}(\mathbb{G}, q, g, g_1, g_2, g_3)$

▷ $\mathcal{D}$ succeeds if $r = b$

DDH hard w.r.t. Gen $\mathcal{G}$ if any ppt $\mathcal{D}$ succeeds only negligibly better than $1/2$.

$\leadsto$ DDH is easy in $\langle \mathbb{Z}_p^*, \cdot, 1 \rangle$

use $\langle QR_p, \cdot, 1 \rangle$ instead.

Abbreviations:

- OWF = one-way function (family/collection)
- OWP = one-way permutation (family/collection)
- TDP = trapdoor one-way permutation
- PRG = pseudorandom generator
- PRF = pseudorandom function
- PRP = pseudorandom permutation
- UOWHF = universal one-way hash function (family/collection)
- CRHF = collision resistant hash function (family/collection)
- ES = (private-key) encryption scheme
- PKES = public-key encryption scheme
- MAC = message authentication code
- DSS = digital signature scheme
- DLP = discrete logarithm problem