Technische Universität München (I7) **Summer term 2016**
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Computational Complexity – Homework 6

Discussed on 30.05.2016.

Exercise 6.1

You have seen that 2SAT is in **NL**. Show that 2SAT is also **NL**-hard.

Exercise 6.2

Show that deciding the inequivalence of context-free grammars over one-letter terminal alphabet is Σ_2^p-hard. You can make use of Σ_2^p-hardness of integer expression inequivalence.

What does it imply for the equivalence problem?

Exercise 6.3

Under the assumption that $3\textsc{Sat} \leq_p \overline{3\textsc{Sat}}$ show that $\mathbf{NP} = \mathbf{PH}$.

Exercise 6.4

Apart from the certificate definition and the alternative bounded alternating Turing machine characterization, there is one more standard characterization of the polynomial hierarchy via *oracles*.

For a language L, an oracle machine M^L is a Turing machine which can moreover do the following kind of computation steps. It can write down a word w on a special tape and ask whether $w \in L$ and it immediately receives the correct answer. One can also talk about this machine even when the oracle is not specified, then we write $M^?$.

Example: In Exercise 3.4 (a), you have constructed an example of M^{SAT} where $M^?$ is a polynomial time TM.

- Prove or disprove: for every $M^?$, if $A \subseteq B$ then $\mathcal{L}(M^A) \subseteq \mathcal{L}(M^B)$.
- Prove or disprove: if $A \subseteq B$ then $\mathbf{P}^A \subseteq \mathbf{P}^B$ (as classes).

The polynomial hierarchy can be defined inductively setting $\Sigma_0^p = \Pi_0^p = \mathbf{P}$ and

$$\Sigma_{i+1}^p = \mathbf{NP}^{\Sigma_i^p}$$

$$\Pi_{i+1}^p = \textbf{co-NP}^{\Sigma_i^p}$$

where A^B is the set of decision problems solvable by a Turing machine in class A with an oracle for some complete problem in class B.

- Show this yields the same hierarchy as the original definition.

One can also define $\Delta_{i+1}^p = \mathbf{P}^{\Sigma_i^p}$ and show that $\Delta_{i+1}^p \subseteq \Sigma_{i+1}^p \cap \Pi_{i+1}^p$ and it contains all languages expressible as Boolean combinations (unions, intersections, complements) of languages of Σ_i^p and Π_i^p.

- What is the relationship of these classes to $\mathbf{DP} = \{L \mid \exists M, N \in \mathbf{NP} : L = M \setminus N\}$?