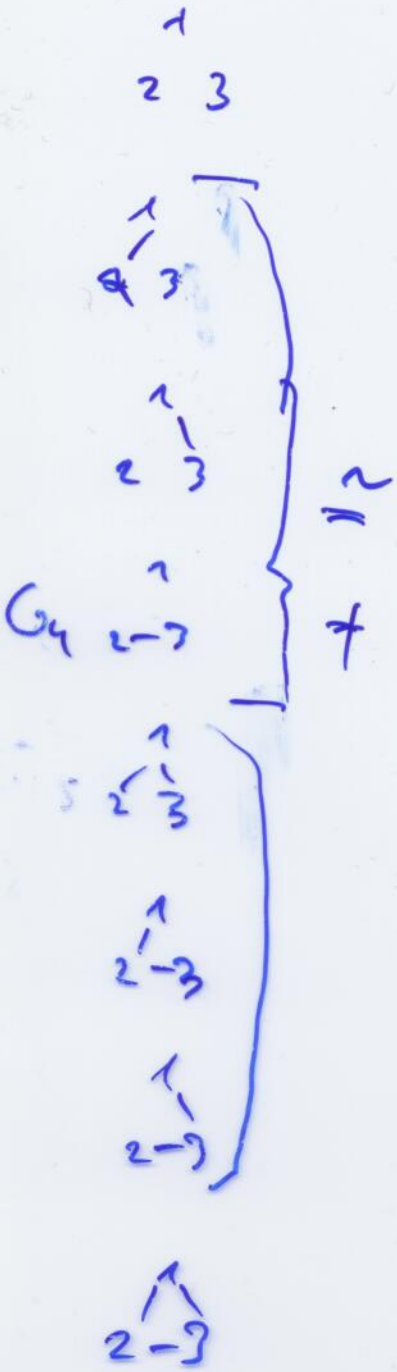


Permutations

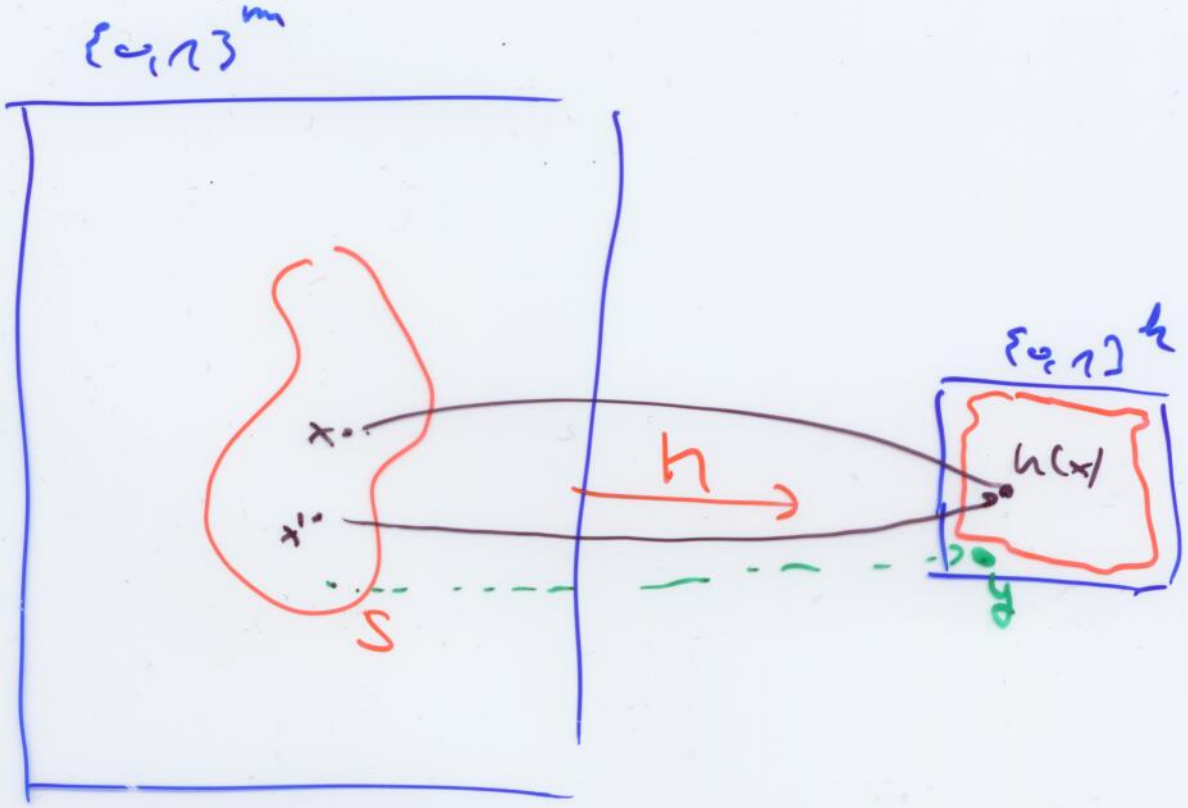
	1	2	3
π_1	1	2	3
π_2	1	3	2
π_3	2	1	3
π_4	2	3	1
π_5	3	1	2
π_6	3	2	1

$\pi_2(G_4) = G_4$

S for G_4, G_5

$\{ \{G_2\} \rightarrow \{\pi_1, \pi_3\}$
$\{G_3\} \rightarrow \{\pi_1, \pi_6\}$
$\{G_4\} \rightarrow \{\pi_1, \pi_2\}$
$\{G_5\} \rightarrow \{\pi_1, \pi_2\}$
$\{G_6\} \rightarrow \{\pi_1,$
$\{G_2\} \rightarrow \{\pi_1,$ $\}\}$

$|S| = 12 = 2 \cdot 3!$


$\{0,1\}^m$
$\{0,1\}^k$
x
x'
h
h(x)
S
y

3|

show: $\Sigma_2^p = \Pi_2^p \Leftarrow \Sigma_2^p \subseteq \Pi_2^p$

$\Leftarrow \Sigma_2 SAT \in \Pi_2^p$ /

true $\psi = \exists x \in \{0,1\}^n \forall y \in \{0,1\}^n F(x,y) \in \Pi_2$

$\Leftrightarrow \exists x \in \{0,1\}^n. f'(F) \in GNI$

| GI is NP-complete

$\downarrow$

GNI is coNP-complete

$\downarrow$ $\exists$ red. f from TAUT

for every φ

$\forall \vec{y}\ \varphi(y)$ true $\Leftrightarrow f(\varphi) \in GNI$ $\langle G_1, G_2 \rangle$

| GNI $\in AM[2]$ with perfect completeness and soundness error $< 2^{-n}$

$\downarrow$

(G_1, G_2)

$V \xrightarrow{r \in \{0,1\}^m} P$

$\xleftarrow{a \in \{0,1\}^m}$

$\neq$ $\Pr[V(G_1, G_2, r, a) = 1] = 1$ (1, 2: $\Leftarrow 1$)

$\leq$ ... $< 2^{-n}$

$\forall r \in \{0,1\}^m. \exists x \in \{0,1\}^n \exists a \in \{0,1\}^m!$. $V(f'(x), r, a) = 1$ /

$\downarrow$ decidable in Π_2^p

4)

1) follows from perfect completeness

2) $\exists x \in \{0,1\}^n . f'(F) \in GNI$ is false

$\Leftrightarrow \forall x \in \{0,1\}^n . f'(F) \notin GNI$

$\Rightarrow \nexists r \in \{0,1\}^m \forall x \in \{0,1\}^n \forall a \in \{0,1\}^m . V(f'(x), r, a) = 0$

soundness error $\square$