$\triangleleft$

Assume $P = NP$

show: $\forall i \geq 1 \quad \Sigma_i^P, \Pi_i^P \subseteq P$

Base case $i = 1$: $\Sigma_1^P = NP$, $\Pi_1^P = coNP$

Base case follows by assumption

Step: $i-1 \mapsto i$:

Let $L \in \Sigma_i^P$

$\Rightarrow \exists$ TM (det., poly) M and polyn. q

$$x \in L \Leftrightarrow \exists u_1 \in \{0,1\}^{q(|x|)} \underbrace{\forall u_2 \in \{0,1\}^{q(|x|)} \ldots Q_i u_i \in \{0,1\}^{q(|x|)}}_{\varphi} M(x, u_1, \ldots u_i) = 1 \quad (*)$$

Define $\langle x, u_1 \rangle \in L' \Leftrightarrow \varphi \, . \, M(x, u_1 \ldots u_i) = 1$

$\Rightarrow L' \in \Pi_{i-1}^P$

$\underset{\text{I.H.}}{\Rightarrow} L' \in P$

$\underset{\text{Def. P}}{\Rightarrow} \exists$ poly-time, det TM M' s.t. $L(M') = L'$

$x \in L \Leftrightarrow \exists u_1 \in \{0,1\}^{q(|x|)} . M'(x, u_1) = 1$

$\Rightarrow L \in NP$

$\underset{\text{ass.}}{\Rightarrow} L \in P$ $\square$

$i = \infty$ PSPACE QBF