Complexity Theory – Homework 7

Discussed on 09.06.2010.

Exercise 7.1

(a) Show that **RP** does not change if we replace in the definition $\geq 3/4$ by $\geq n^{-k}$ or $\geq 1 - 2^{-n^k}$ (with $k > 0$).

(b) Let $L \in \mathbf{NP}$ be decided by a poly-time TM $M(x, u)$ with certificates u of length $p(|x|)$.

Prove or disprove that $x \in L \Rightarrow \Pr[A_{M,x}] \geq n^{-k}$ needs to hold for some $k > 0$ if a polynomial number $r(|x|)$ of reruns should suffice to reduce the probability of false negatives below any given bound $c \in (0, 1)$.

Remark: Use that $(1 - 1/k)^k \approx e^{-1}$ for large k.

Exercise 7.2

Show that, if $\mathbf{NP} \subseteq \mathbf{BPP}$, then $\mathbf{RP} = \mathbf{NP}$.

Exercise 7.3

Show that $L \in \mathbf{ZPP}$ if and only if L is decided by some PTM in expected polynomial time.

Exercise 7.4

Show that

(a) **RP**, **BPP**, and **PP** are closed under $\leq_p$.

Remark: Recall that a class $\mathbf{C}$ is closed under $\leq_p$ if $A \leq_p B \wedge B \in \mathbf{C} \Rightarrow A \in \mathbf{C}$.

(b) **RP** and **BPP** are closed under intersection and union.

Exercise 7.5

For a given $c > 0$ let a language L be in $\mathbf{PP}_{\geq c}$ if $x \in L \Leftrightarrow \Pr[A_{M,x}] \geq c$. Similarly, the class $\mathbf{PP}_{>c}$ is defined.

Show that

(a) $\mathbf{PP}_{>1/2} = \mathbf{PP}_{\geq 1/2}$.

(b) $\mathbf{PP}_{>1/2}$ is closed under complement and symmetric difference.

Remark: The symmetric difference $A \Delta B$ of two sets A, B is defined by $A \Delta B := (A \setminus B) \cup (B \setminus A)$.

(c) MajSat is $\mathbf{PP}_{>1/2}$-complete.

Remark: MajSat is the following problem: Given a Boolean expression with n variables, is it true that the majority of the 2^n truth assignments to its variables, i.e., at least $2^{n-1} + 1$ of them, satisfy it?

*(d) $\mathbf{PP} = \mathbf{PP}_{\geq 1/2}$.

Remark: Recall that we defined $\mathbf{PP} := \mathbf{PP}_{\geq 3/4}$.