Complexity Theory – Homework 6

Discussed on 02.06.2010.

Exercise 6.1

You have seen in the lecture the characterisation of **NL** using log-space TMs which besides the input x have also access to a certificate u of length polynomial in $|x|$ with the restriction that u must be read from left to right (read-once).

(a) Assume we relax the read-once restriction to: between reading u_i and u_{i+1} at most the bits $u_{i-k\log(|x|)} \dots u_i$ can be read (for some fixed $k \in \mathbb{N}$).

Does this make the TM more powerful?

(b) Assume now that we drop the read-once restriction completely.

Show that then 3Sat can be decided.

Conclusion: One can show that both the Cook-Levin-reduction and the reduction of Sat to 3Sat works also in log-space. Without the read-once restriction we therefore move from **NL** to **NP**.

Exercise 6.2

Under the assumption that $3\textsc{Sat} \leq_p \overline{3\textsc{Sat}}$ show that $\mathbf{NP} = \mathbf{PH}$.

Exercise 6.3

The class **DP** is defined as follows:

$$\mathbf{DP} := \{L_1 \cap L_2 \mid L_1 \in \mathbf{NP}, L_2 \in \mathrm{co}\mathbf{NP}\}.$$

Show that:

(a) Exact Indset $\in$ **DP**.

Reminder: Exact Indset $:= \{\langle G, k\rangle \mid$ the largest indepedent set of G has size exactly $k\}$.

(b) $\mathbf{NP} \cup \mathrm{co}\mathbf{NP} \subseteq \mathbf{DP} \subseteq \mathbf{\Sigma}_2^p \cap \mathbf{\Pi}_2^p$.

(c) The following language is **DP**-complete w.r.t. $\leq_p$:

$$3\textsc{Sat}\text{-}3\textsc{Unsat} := \{(\phi, \psi) \mid \phi, \psi \text{ are 3CNF-formula with } \phi \text{ satisfiable, } \psi \text{ unsatisfiable }\}.$$

*(d) 3Sat-3Unsat $\leq_p$ Exact Indset.

Remark: Let $f(\phi) = (G_\phi, m_\phi)$ be the poly-time reduction from 3Sat to Indset. Check that m_ϕ is both the number of clauses of ϕ and the least upper bound on the size of any independent set in G_ϕ by definition of f.