Automata and Formal Languages

Winter 2019/20

Syllabus

Course schedule

Lectures

Jan Kretinsky

Monday:	10:00	11:30	Room:	IMETUM, E.126
Tuesday:	10:00	11:30	Room:	IMETUM, E.126

Exercises

Marijana Lazic and Chana Weil-Kennedy

Thursday:	14:00	15:30	Room:	02.13.010

Course schedule

Lectures

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Monday:	10:15 11:45?	Room:	IMETUM, E.126
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Exercises

Marijana Lazic and Chana Weil-Kennedy

Thursday:	14:15 15:45?	Room:	02.13.010

Course content

Automata on finite words

1. Automata classes and conversions
2. Minimization and reduction
3. Boolean operations and tests
4. Operations on relations
5. Operations on finite universes: decision diagrams
6. Automata and logic
7. Pattern-matching, verification, Presburger arithmetic

Course content

Automata on infinite words

8. Automata classes and conversions
9. Boolean operations
10. Emptiness check
11. Verification using temporal logic

Material

- Lecture notes available online
- Slides available online
- No book to buy

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www7.in.tum.de > Teaching
               > Automata
               > more info
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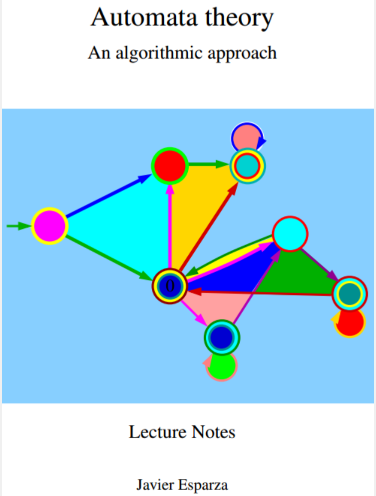

Automata theory: brief recap

Formal languages

An *alphabet* is a nonempty finite set of *letters*

e.g. $\{0, 1\}$, $\{a, b, \ldots, z\}$, $\{\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]\}$, $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$

A *(finite) word* is a finite sequence of letters

e.g. 1001, $\texttt{hello}$, $\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$, $\clubsuit\clubsuit\diamondsuit$, ε

A *language* is a set of words

e.g. $\{1, 10, 100, 1000, \ldots\}$, $\{aa, aba, abbba, \ldots\}$

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Formal languages

Let $u = a_1 \cdots a_n$ and $v = b_1 \cdots b_m$ be words

$$\begin{aligned} \textit{Concatenation}: \quad u \cdot v &= uv = a_1 \cdots a_n b_1 \cdots b_m \\ \varepsilon \cdot u &= u = u \cdot \varepsilon \end{aligned}$$

Exponentiation: $u^0 = \varepsilon$, $u^{k+1} = u^k \cdot u$

e.g. $\mathtt{a}^0 = \varepsilon$, $\mathtt{a}^1 = \mathtt{a}$, $(\mathtt{hallo})^2 = \mathtt{hallohallo}$,
$\mathtt{1}^5 = \mathtt{11111}$, $\varepsilon^{1000} = \varepsilon$, $\mathtt{ab} \cdot \mathtt{cde} = \mathtt{abcde}$

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Let L and L' be languages over alphabet Σ

Concatenation: $L \cdot L' \;=\; LL' \;=\; \{u \cdot v : u \in L, v \in L'\}$

Exponentiation: $L^0 = \{\varepsilon\},\ L^{k+1} = L^k \cdot L$

Iteration: $L^* \;=\; L^0 \;\cup\; L^1 \;\cup\; L^2 \;\cup\; \cdots$

Complement: $\overline{L} = \Sigma^* \setminus L$

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$$\begin{aligned} \{\mathtt{aa}, \mathtt{bb}\} \cdot \{\mathtt{c}, \mathtt{d}\} &= \{\mathtt{aac}, \mathtt{aad}, \mathtt{bbc}, \mathtt{bbd}\}, \\ \{\mathtt{aa}, \mathtt{b}\}^* &= \{\varepsilon, \mathtt{aa}, \mathtt{b}, \mathtt{aaaa}, \mathtt{aab}, \ldots\} \end{aligned}$$

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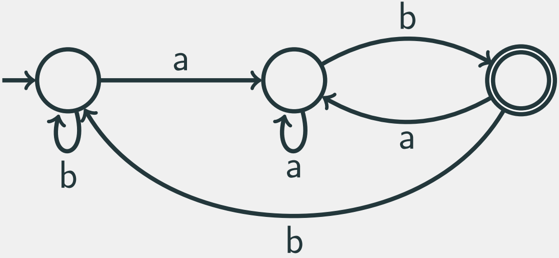
Deterministic finite automata (DFA)

- *States*: nonempty finite set Q
- *Alphabet*: nonempty finite set Σ
- *Transitions*: $\delta : Q \times \Sigma \to Q$
- *Initial state*: $q_0 \in Q$
- *Final states*: $F \subseteq Q$



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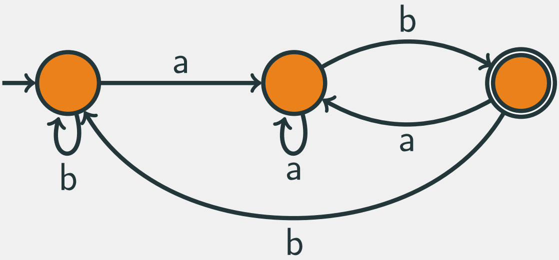

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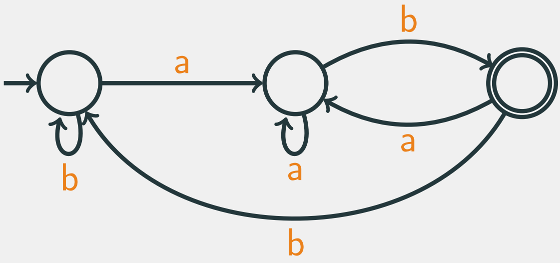

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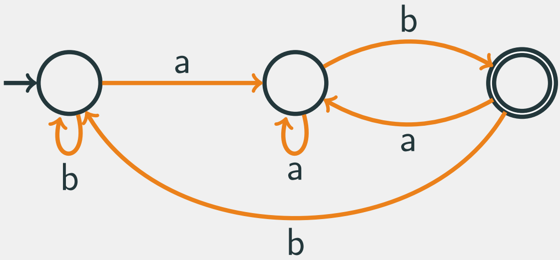

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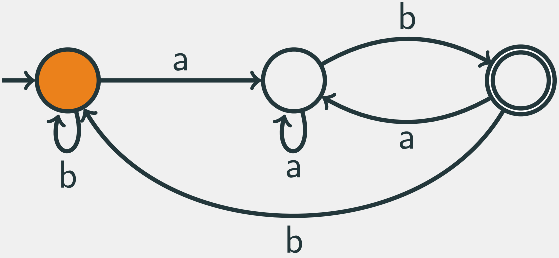

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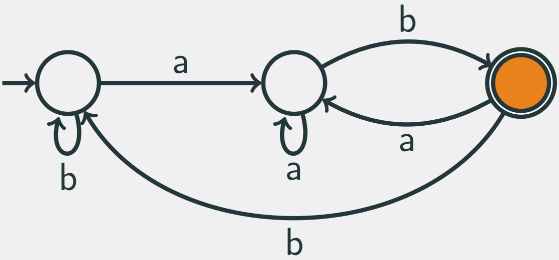
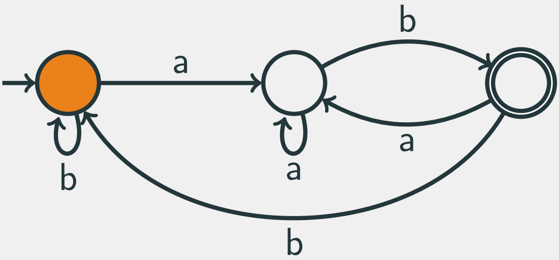


Deterministic finite automata (DFA)

$$w = aabab$$



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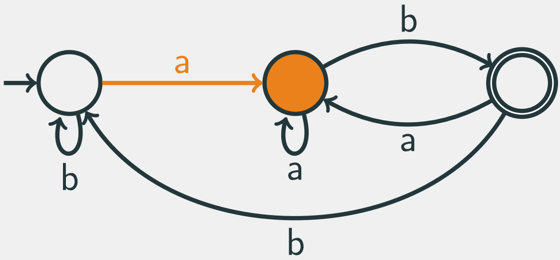

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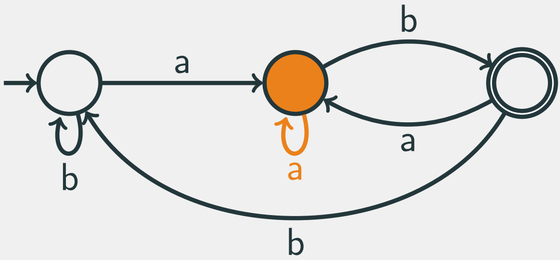

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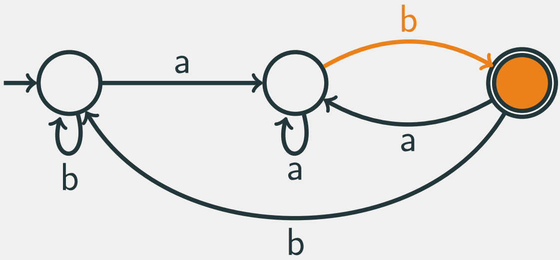

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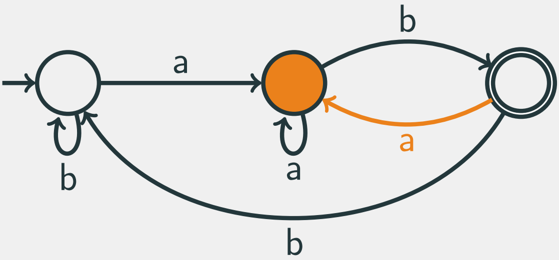

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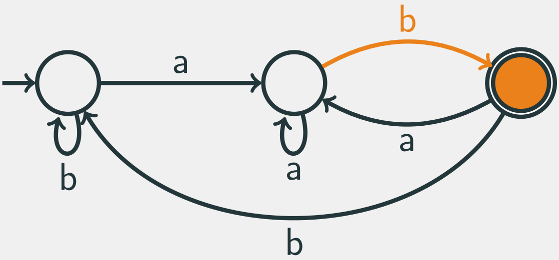
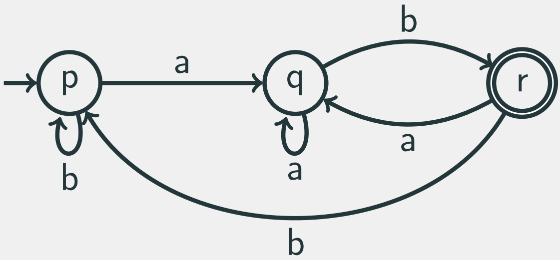


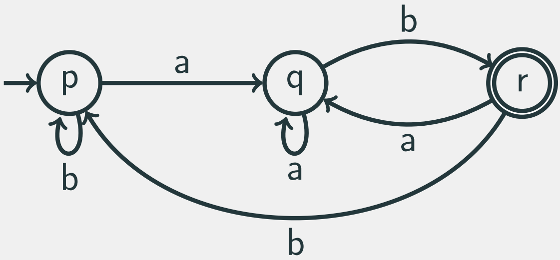
Deterministic finite automata (DFA)

$$p \xrightarrow{\texttt{a}} q \xrightarrow{\texttt{a}} q \xrightarrow{\texttt{b}} r \xrightarrow{\texttt{a}} q \xrightarrow{\texttt{b}} r$$

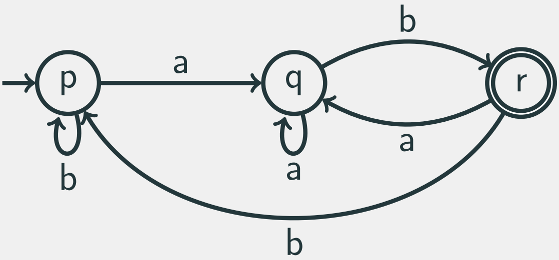


Deterministic finite automata (DFA)

$$p \xrightarrow{\texttt{aabab}} r$$



$$L(A) = \{w \in \Sigma^* : \exists q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



Deterministic finite automata (DFA)

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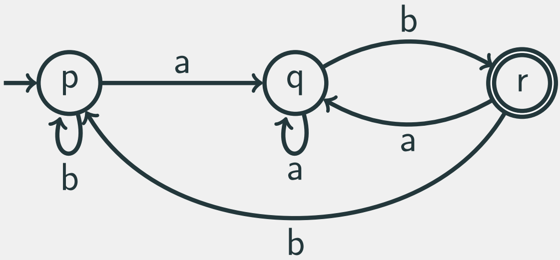
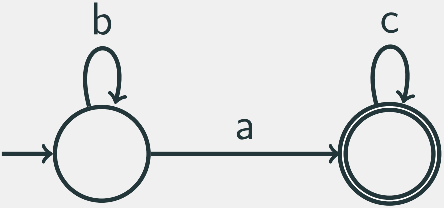


Transition function δ defined *on every* input



DFA: trap states and unreachable states

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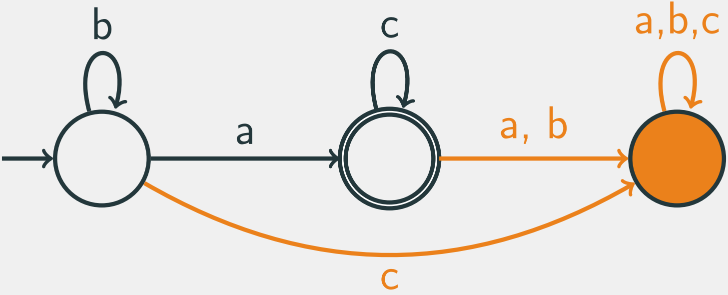
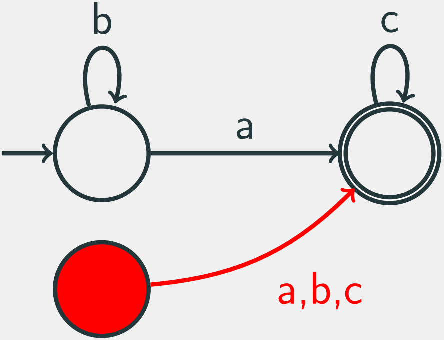


DFA: trap states and unreachable states

Every state *reachable* from initial state



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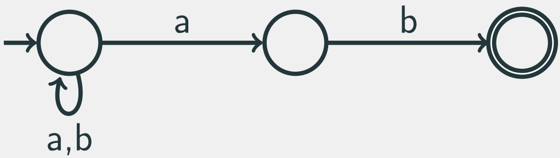

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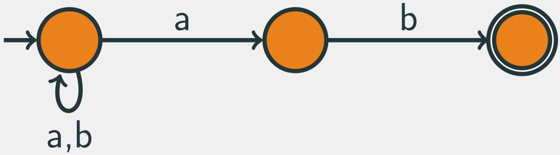

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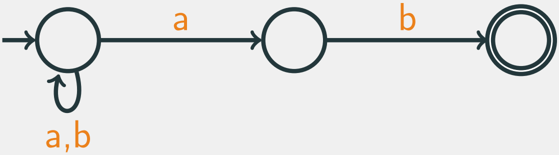

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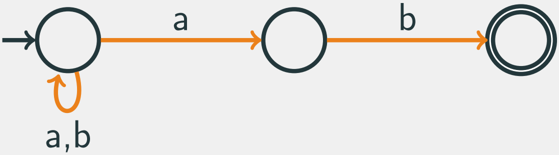

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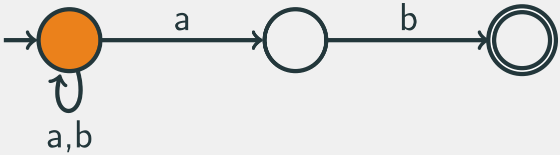

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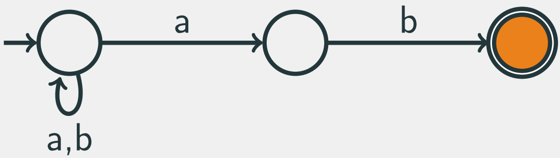

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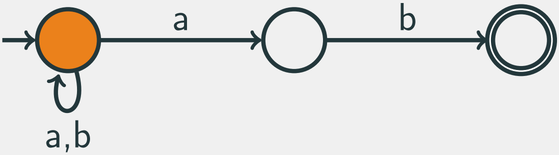

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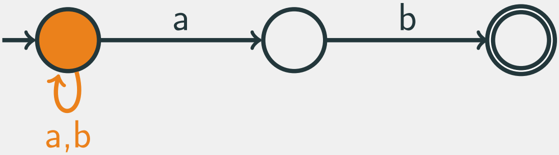

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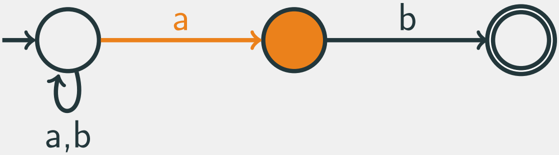

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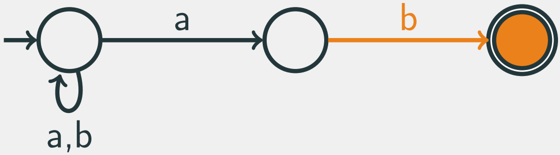

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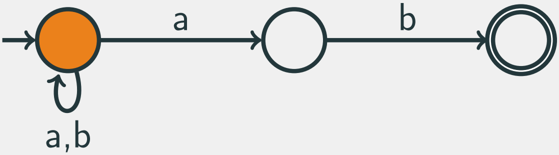

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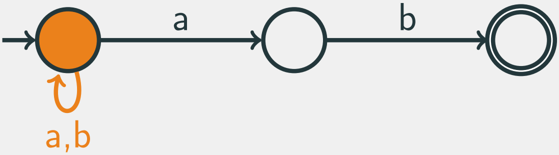

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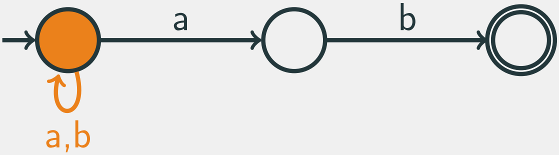

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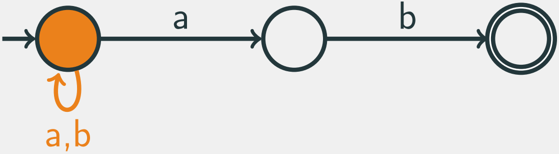
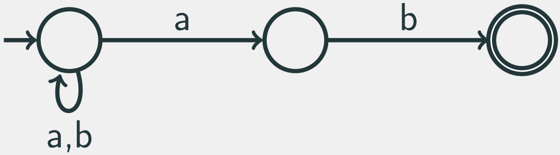


Nondeterministic finite automata (NFA)

$$p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} p_n$$

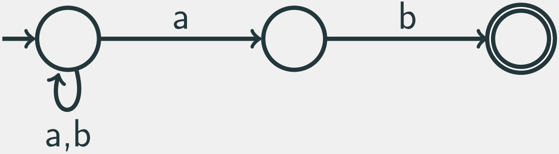
$$\Updownarrow$$

$$p_i \in \delta(p_{i-1}, a_i) \text{ for every } 0 < i \leq n$$



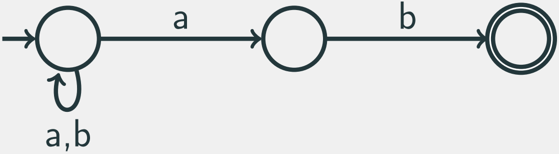
Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \exists q_0 \in Q_0, q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \quad w \text{ ends with } \texttt{ab} \quad \}$$



Regular expressions

$$r \quad ::= \quad \emptyset \quad | \quad \varepsilon \quad | \quad a \quad | \quad r_1\, r_2 \quad | \quad r_1 + r_2 \quad | \quad r^*$$

Regular expressions

$$r \;::=\; \emptyset \;\mid\; \varepsilon \;\mid\; a \;\mid\; r_1\, r_2 \;\mid\; r_1 + r_2 \;\mid\; r^*$$

$$\begin{aligned}
L(\emptyset) &= \emptyset & \qquad L(r_1\, r_2) &= L(r_1) \cdot L(r_2) \\
L(\varepsilon) &= \{\varepsilon\} & \qquad L(r_1 + r_2) &= L(r_1) \cup L(r_2) \\
L(a) &= \{a\} & \qquad L(r^*) &= L(r)^*
\end{aligned}$$

Regular expressions

$$L(\,(a+b)^*ab\,) = \{w \in \{a,b\}^* : w \text{ ends with } \texttt{ab}\}$$

$$\begin{array}{lcl} L(\emptyset) & = & \emptyset \\ L(\varepsilon) & = & \{\varepsilon\} \\ L(a) & = & \{a\} \end{array} \qquad \begin{array}{lcl} L(r_1\, r_2) & = & L(r_1) \cdot L(r_2) \\ L(r_1 + r_2) & = & L(r_1) \cup L(r_2) \\ L(r^*) & = & L(r)^* \end{array}$$

More examples

$$L = \{w \in \{\texttt{a}, \texttt{b}\}^* : w \text{ contains } \texttt{aaa}\}$$

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Regular expression?

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Regular expression?

$$(\mathtt{a} + \mathtt{b})^* \mathtt{aaa} (\mathtt{a} + \mathtt{b})^*$$

More examples

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NFA?

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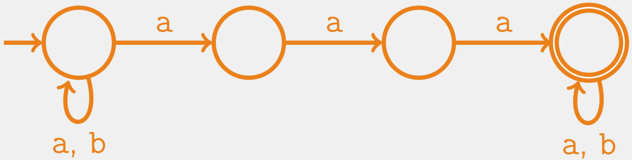
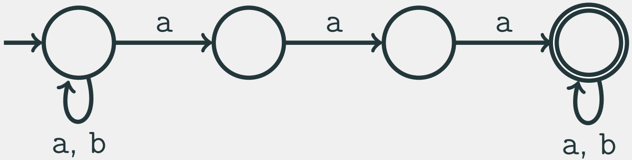


$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : w \text{ contains aaa}\}$$

DFA?



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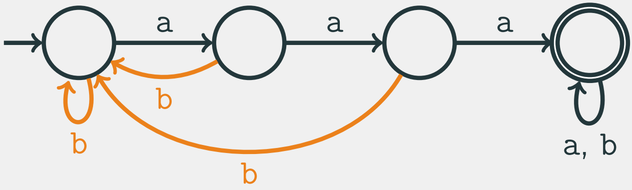

More examples

$$L = \{w \in \{0,1\}^* : w \text{ contains an even number of 0 or an odd number of 1}\}$$

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Regular expression?

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Regular expression?

$$(1^*01^*0)^*1^* + (0^*10^*1)^*0^*10^*$$

More examples

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NFA?

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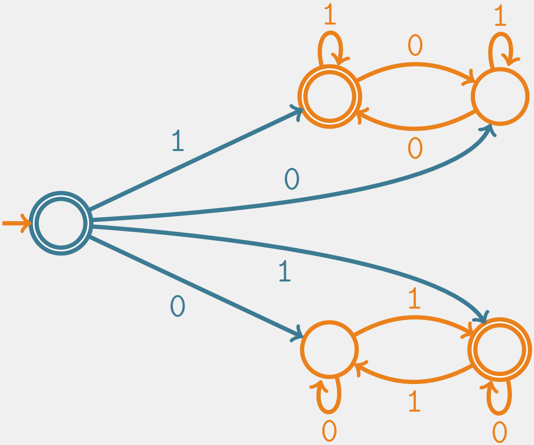

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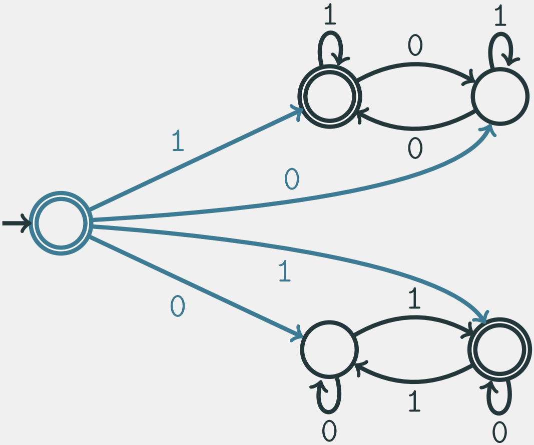

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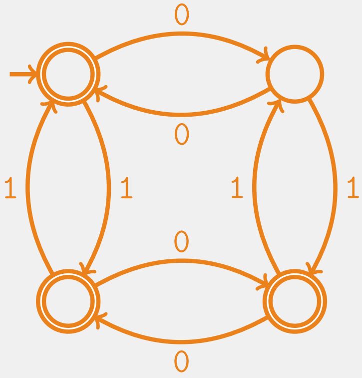
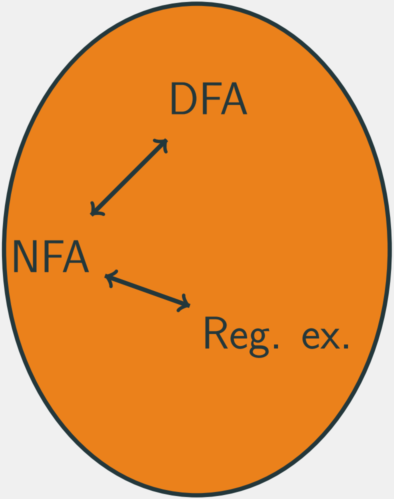


Regular languages



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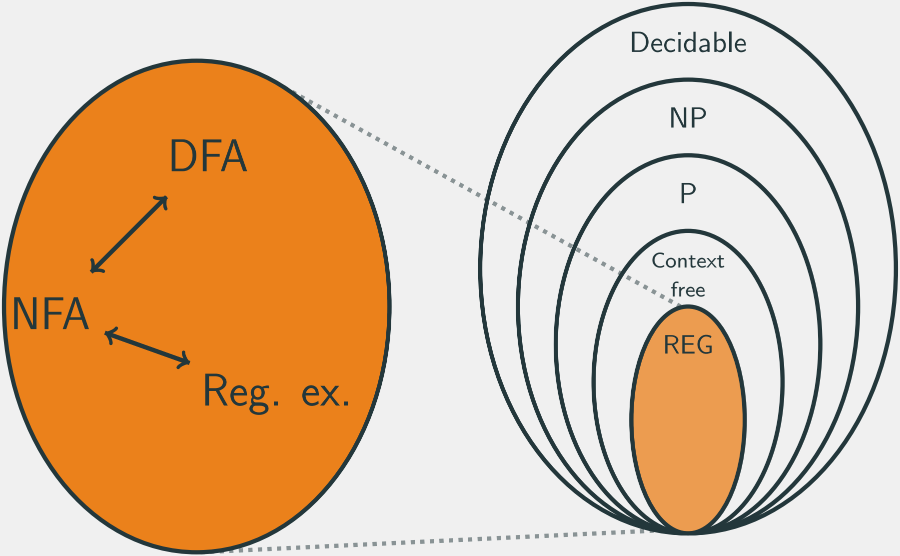

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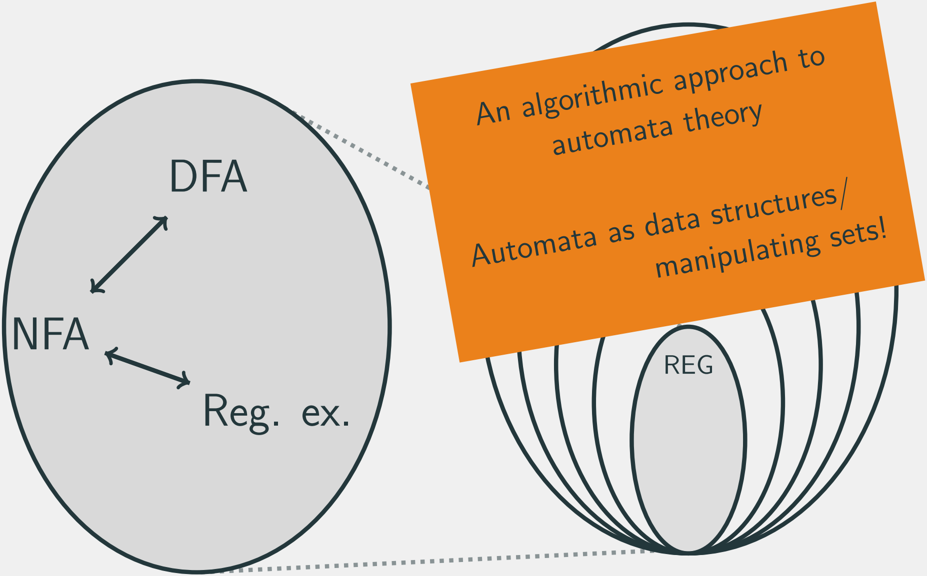
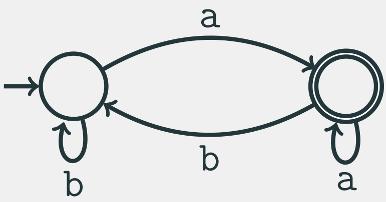


Beyond finite words

Büchi automata

An *infinite word* is an infinite sequence $a_0 a_1 a_2 \cdots$ over some Σ

A *Büchi automaton* is "as an NFA", but accepts infinite words



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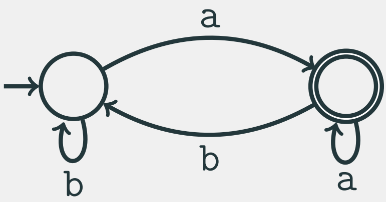

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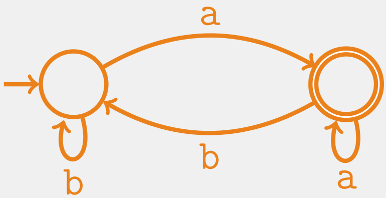

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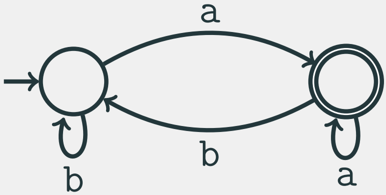


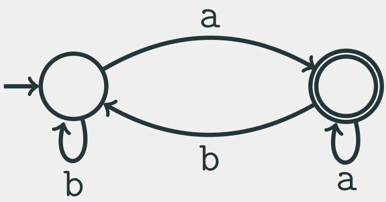
$L_\omega(A) = \{w \in \{\mathtt{a}, \mathtt{b}\}^\omega : w \text{ contains infinitely many } \mathtt{a}\}$

Büchi automata

An *i* … r some Σ

Coming later this semester!

A *Bü* … te words



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