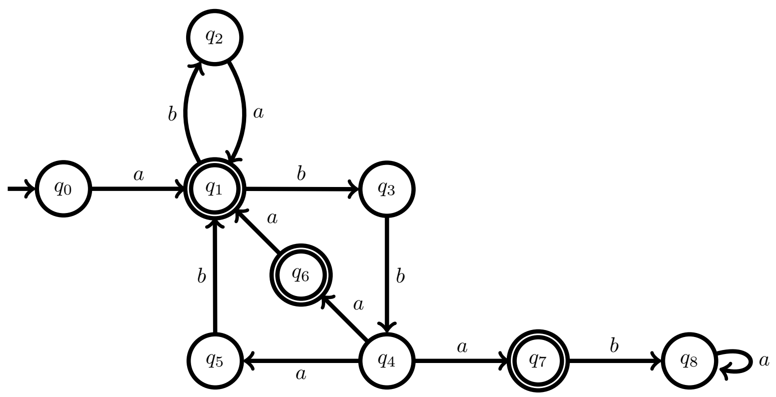
Automata and Formal Languages — Exercise Sheet 15

Exercise 15.1
Let B be the following Büchi automaton:



(a) Execute the emptiness algorithm *TwoStack* on B.

(b) Which lassos of B can be found by *TwoStack*?

Exercise 15.2
Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give LTL formulas for the following ω-languages:

(a) $\{p,q\}\,\emptyset\,\Sigma^\omega$

(b) $\Sigma^*\,\{q\}^\omega$

(c) $\Sigma^*\,(\{p\} + \{p,q\})\,\Sigma^*\,\{q\}\,\Sigma^\omega$

(d) $\{p\}^*\,\{q\}^*\,\emptyset^\omega$

Exercise 15.3
Let $\mathrm{AP} = \{p, q, r\}$. Give formulas for the computations satisfying the following properties:

(a) if q eventually holds, then p must not hold before q first does.

(b) if q eventually holds, then p holds at some point before q first holds.

(c) p always holds everywhere between q and r.

(d) p, and only p, holds at even positions and q, and only q, holds at odd positions.

Exercise 15.4

Let AP $= \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give Büchi automata for the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

(b) $(\mathbf{GF}p) \rightarrow (\mathbf{F}q)$

(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p\ \mathbf{U}\ (p \rightarrow q))$

(e) $\mathbf{F}q \rightarrow (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

Solution 15.1

(a) Let us assume that the algorithms always pick states in ascending order with respect to their indices. The algorithm reports "non empty" after the following execution:

$\xrightarrow{C.\text{push}(q_0),\ V.\text{push}(q_0)}$

C	V
q_0	q_0

$\xrightarrow{C.\text{push}(q_1),\ V.\text{push}(q_1)}$

C	V
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_2),\ V.\text{push}(q_2)}$

C	V
q_2	q_2
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_2
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_2
	q_1
q_0	q_0

(b) All of them. The lasso q_0, q_1, q_2, q_1 is found by the above execution. The lasso $q_0, q_1, q_3, q_4, q_6, q_1$ is found by the following execution:

$\xrightarrow{C.\text{push}(q_0),\ V.\text{push}(q_0)}$

C	V
q_0	q_0

$\xrightarrow{C.\text{push}(q_1),\ V.\text{push}(q_1)}$

C	V
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_3),\ V.\text{push}(q_3)}$

C	V
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_4),\ V.\text{push}(q_4)}$

C	V
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_6),\ V.\text{push}(q_6)}$

C	V
q_6	q_6
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_6
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

The lasso $q_0, q_1, q_3, q_4, q_5, q_1$ is found by the following execution:

$\xrightarrow{C.\text{push}(q_0),\ V.\text{push}(q_0)}$

C	V
q_0	q_0

$\xrightarrow{C.\text{push}(q_1),\ V.\text{push}(q_1)}$

C	V
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_3),\ V.\text{push}(q_3)}$

C	V
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_4),\ V.\text{push}(q_4)}$

C	V
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{push}(q_5),\ V.\text{push}(q_5)}$

C	V
q_5	q_5
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_5
q_4	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_5
	q_4
q_3	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_5
	q_4
	q_3
q_1	q_1
q_0	q_0

$\xrightarrow{C.\text{pop}()}$

C	V
	q_5
	q_4
	q_3
	q_1
q_0	q_0

Solution 15.2

(a) $(p \wedge q) \wedge \mathbf{X}(\neg p \wedge \neg q)$

(b) $\mathbf{FG}(\neg p \wedge q)$

(c) $\mathbf{F}(p \wedge \mathbf{XF}(\neg p \wedge q))$

(d) $(p \wedge \neg q)\ \mathbf{U}\ ((\neg p \wedge q)\ \mathbf{U}\ \mathbf{G}(\neg p \wedge \neg q))$

Solution 15.3

(a) $\mathbf{F}q \rightarrow (\neg p\ \mathbf{U}\ q)$

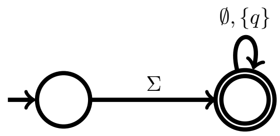
(b) $\mathbf{F}q \rightarrow (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

(c) $\mathbf{G}((q \wedge \mathbf{XF}r) \rightarrow \mathbf{X}(p\ \mathbf{U}\ r))$

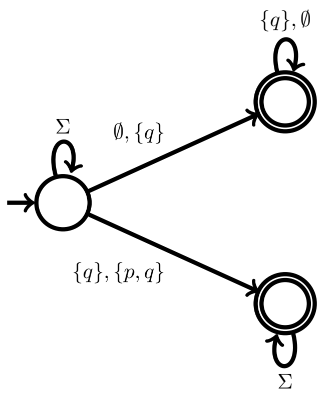
(d) $\mathbf{G}(\neg r) \wedge \mathbf{G}(p \leftrightarrow \neg q) \wedge p \wedge \mathbf{G}(p \rightarrow \mathbf{X}q) \wedge \mathbf{G}(q \rightarrow \mathbf{X}p)$

Solution 15.4

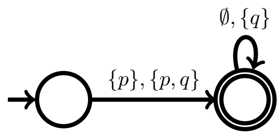
(a)



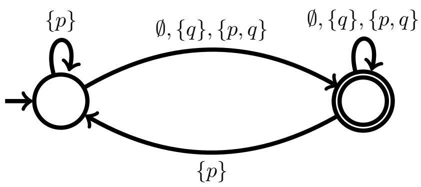
(b) Note that $(\mathbf{GF}p) \to (\mathbf{F}q) \equiv \neg(\mathbf{GF}p) \vee (\mathbf{F}q) \equiv (\mathbf{FG}\neg p) \vee (\mathbf{F}q)$. We construct Büchi automata for $\mathbf{FG}\neg p$ and $\mathbf{F}q$, and take their union:



(c) Note that $p \wedge \neg(\mathbf{XF}p) \equiv p \wedge \mathbf{XG}\neg p$. We construct a Büchi automaton for $p \wedge \mathbf{XG}\neg p$:



(d)



(e)