

Automata and Formal Languages

Winter 2017/18

Syllabus

Course schedule

Lectures

Prof. Javier Esparza (esparza@in.tum.de)

Room: 02.13.010

Monday: 10:00 – 11:30

Thursday: 14:00 – 15:30

Exercises

Dr. Michael Blondin (blondin@in.tum.de)

Room: 02.13.010

Tuesday: 12:00 – 13:30

Course schedule

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Automata on finite words

1. Automata classes and conversions
2. Minimization and reduction
3. Boolean operations and tests
4. Operations on relations
5. Operations on finite universes: decision diagrams
6. Automata and logic
7. Pattern-matching, verification, Presburger arithmetic

Automata on infinite words

8. Automata classes and conversions
9. Boolean operations
10. Emptiness check
11. Verification using temporal logic

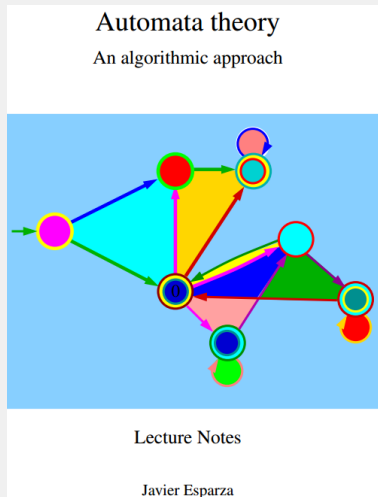
Grading

	Points	Grade
Written exam	[36, 40]	1,0
	[34, 36)	1,3
	[32, 34)	1,7
	[30, 32)	2,0
	[28, 30)	2,3
	[26, 28)	2,7
	[24, 26)	3,0
	[22, 24)	3,3
	[19, 22)	3,7
	[17, 19)	4,0
Exercises not graded!	[11, 17)	4,3
	[5, 11)	4,7
	[0, 5)	5,0

Material

- Lecture notes available online
- Slides available online
- No book to buy

`www7.in.tum.de` > Teaching
> Automata
> more info



Automata theory: brief recap

Formal languages

An *alphabet* is a nonempty finite set of *letters*

e.g. $\{0, 1\}$, $\{a, b, \dots, z\}$, $\{[0], [1], [0], [1]\}$, $\{\clubsuit, \diamond, \heartsuit, \spadesuit\}$

A (*finite*) *word* is a finite sequence of letters

e.g. 1001, hello, $[1][0][1]$, $\clubsuit\clubsuit\diamond$, ϵ

A *language* is a set of words

e.g. $\{1, 10, 100, 1000, \dots\}$, $\{aa, aba, abbba, \dots\}$

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Formal languages

Let $u = a_1 \cdots a_n$ and $v = b_1 \cdots b_m$ be words

Concatenation: $u \cdot v = uv = a_1 \cdots a_n b_1 \cdots b_m$

$$\varepsilon \cdot u = u = u \cdot \varepsilon$$

Exponentiation: $u^0 = \varepsilon, u^{k+1} = u^k \cdot u$

e.g. $a^0 = \varepsilon, a^1 = a, (\text{hallo})^2 = \text{hallohallo},$
 $1^5 = 11111, \varepsilon^{1000} = \varepsilon, ab \cdot cde = abcde$

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Let L and L' be languages over alphabet Σ

Concatenation: $L \cdot L' = LL' = \{u \cdot v : u \in L, v \in L'\}$

Exponentiation: $L^0 = \{\varepsilon\}, L^{k+1} = L^k \cdot L$

Iteration: $L^* = L^0 \cup L^1 \cup L^2 \cup \dots$

Complement: $\bar{L} = \Sigma^* \setminus L$

e.g. $\{aa, bb\} \cdot \{c, d\} = \{aac, aad, bbc, bbd\},$
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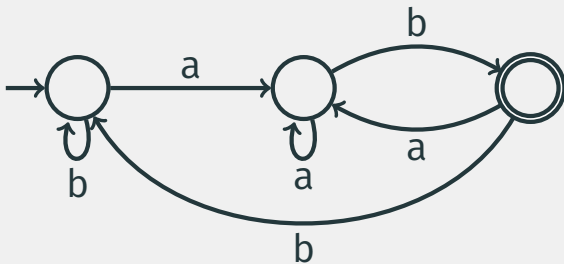
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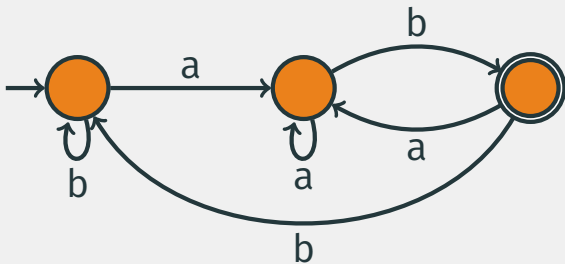
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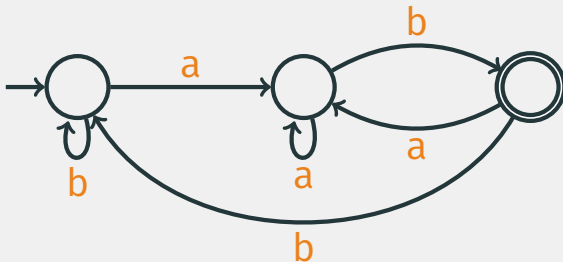
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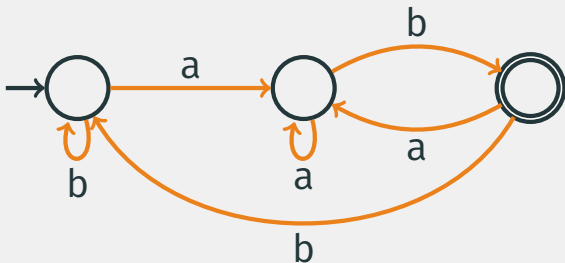
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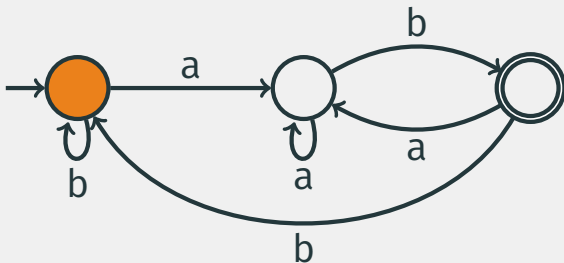
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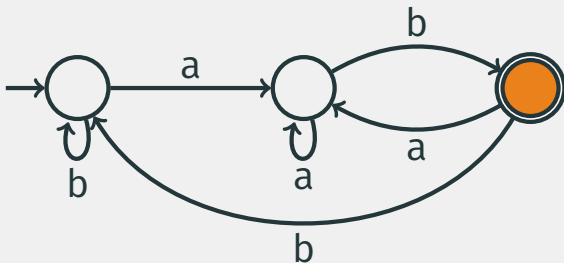
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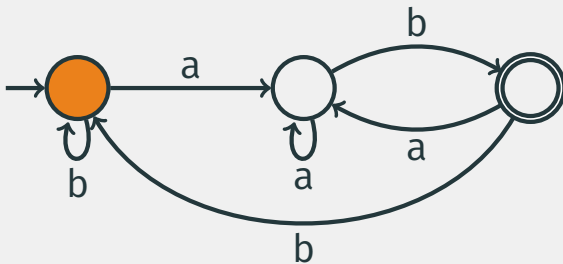
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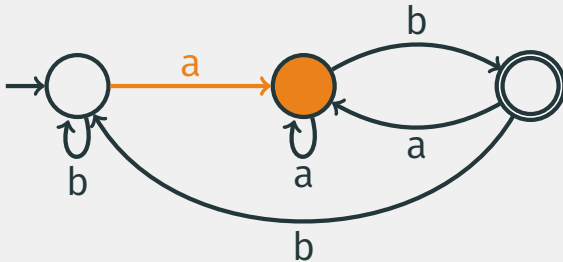
Deterministic finite automata (DFA)

$w = aabab$



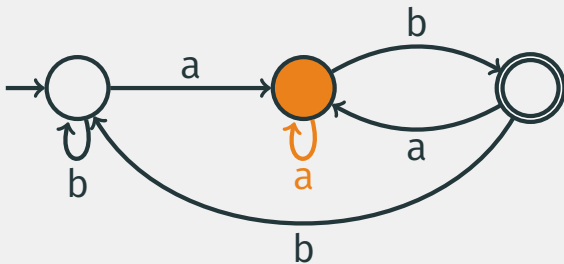
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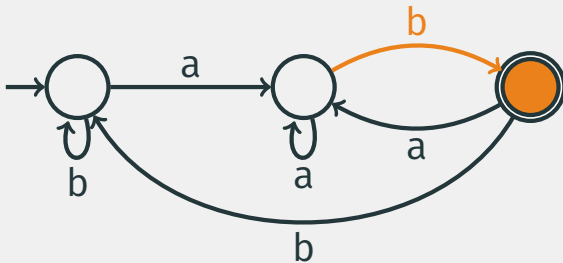
Deterministic finite automata (DFA)

$w = \text{a} \color{orange}{\text{b}} \text{abab}$



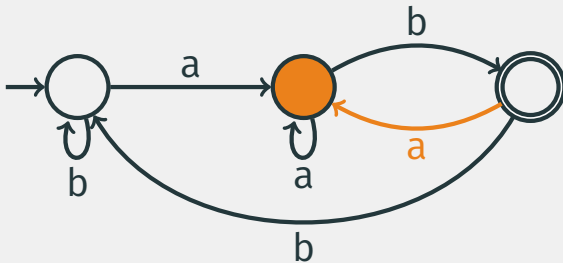
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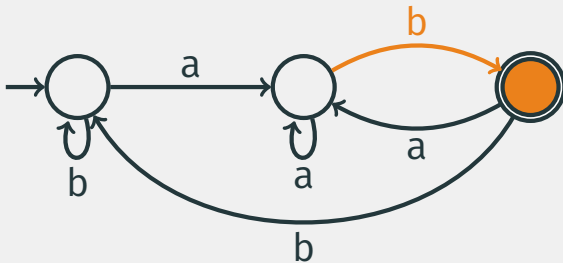
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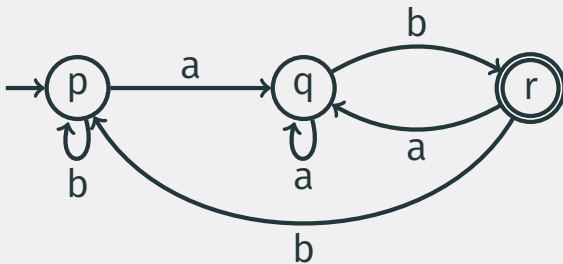
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$w = \text{aaba}b$

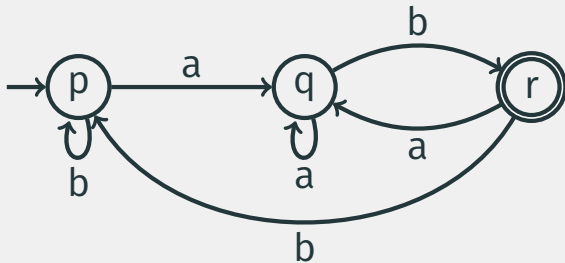


Deterministic finite automata (DFA)

$p \xrightarrow{a} q \xrightarrow{a} q \xrightarrow{b} r \xrightarrow{a} q \xrightarrow{b} r$

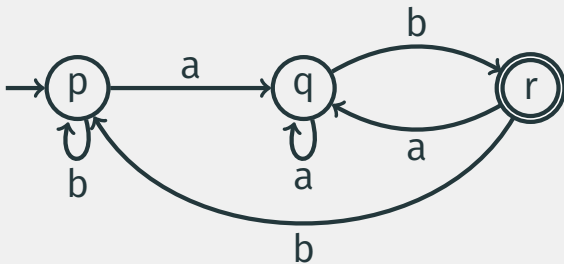


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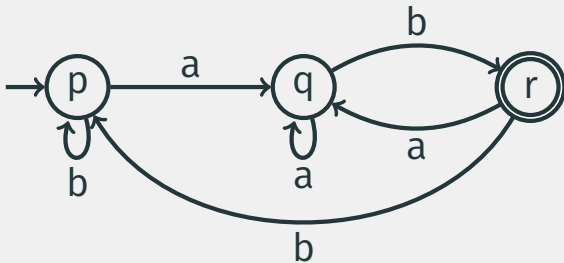
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$$L(A) = \{w \in \Sigma^* : \exists q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



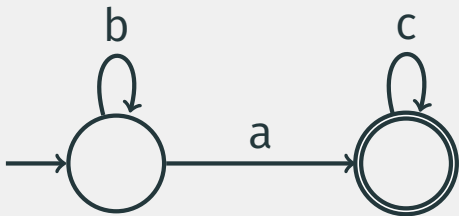
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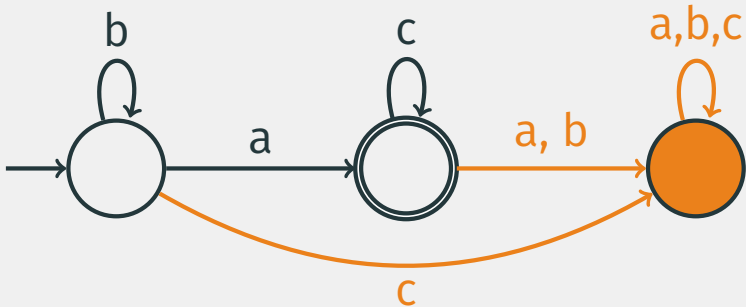
DFA: trap states and unreachable states

Transition function δ defined *on every* input



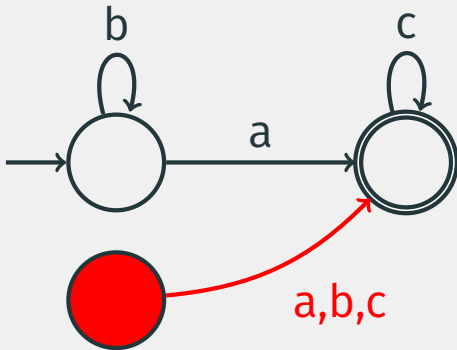
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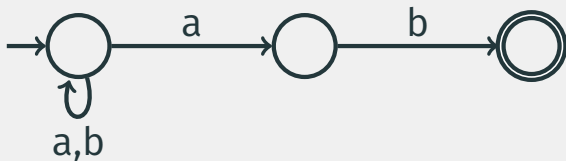
DFA: trap states and **unreachable states**

Every state *reachable* from initial state



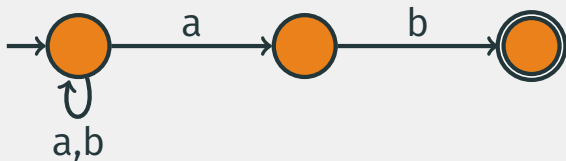
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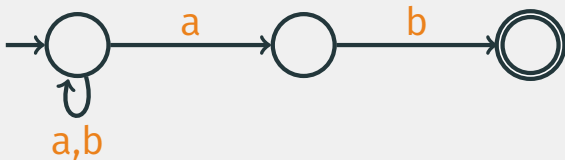
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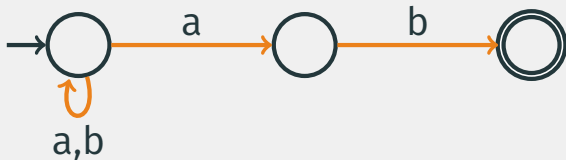
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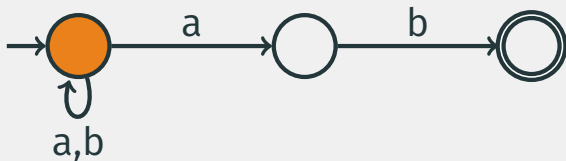
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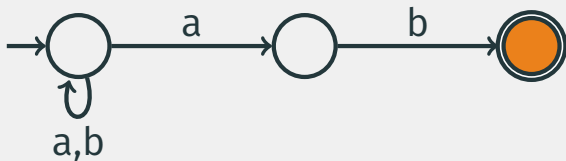
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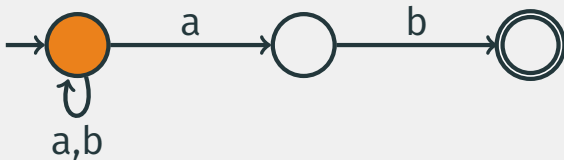
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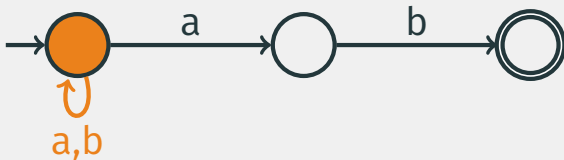
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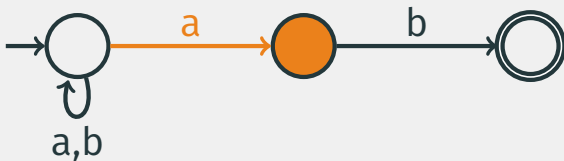
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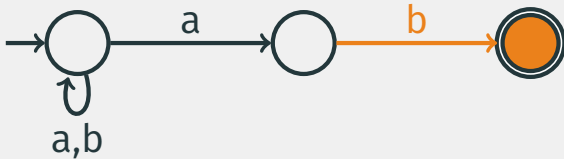
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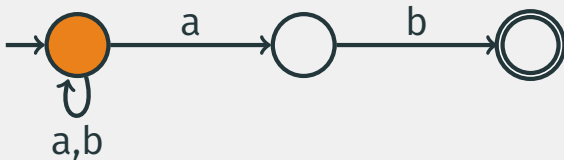
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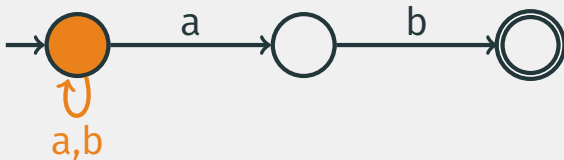
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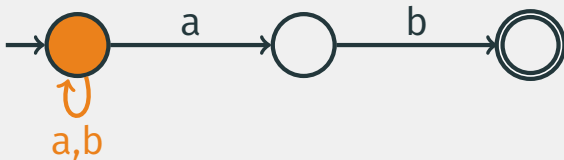
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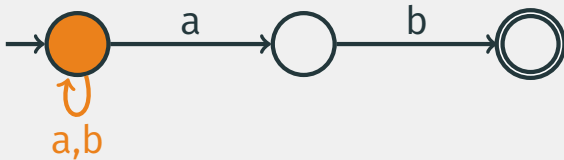
Nondeterministic finite automata (NFA)

$w = \text{a} \text{a} \text{b}$



Nondeterministic finite automata (NFA)

$w = aab$

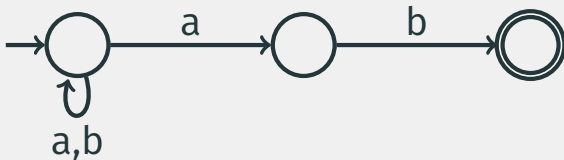


Nondeterministic finite automata (NFA)

$$p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} p_n$$

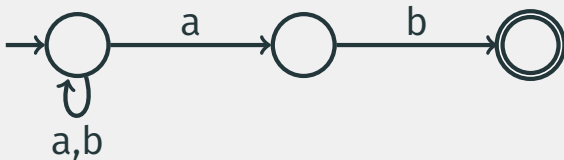


$p_i \in \delta(p_{i-1}, a_i)$ for every $0 < i \leq n$



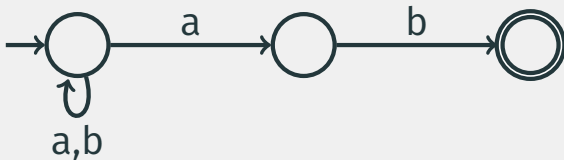
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Regular expressions

$$r ::= \emptyset \mid \varepsilon \mid a \mid r_1 r_2 \mid r_1 + r_2 \mid r^*$$

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$$L(\emptyset) = \emptyset \qquad L(r_1 r_2) = L(r_1) \cdot L(r_2)$$

$$L(\varepsilon) = \{\varepsilon\} \qquad L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(a) = \{a\} \qquad L(r^*) = L(r)^*$$

$$L((a + b)^* ab) = \{w \in \{a, b\}^* : w \text{ ends with } ab\}$$

$$L(\emptyset) = \emptyset \qquad L(r_1 r_2) = L(r_1) \cdot L(r_2)$$

$$L(\varepsilon) = \{\varepsilon\} \qquad L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(a) = \{a\} \qquad L(r^*) = L(r)^*$$

More examples

$$L = \{w \in \{a, b\}^* : w \text{ contains } aaa\}$$

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Regular expression?

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Regular expression?

$$(a + b)^*aaa(a + b)^*$$

More examples

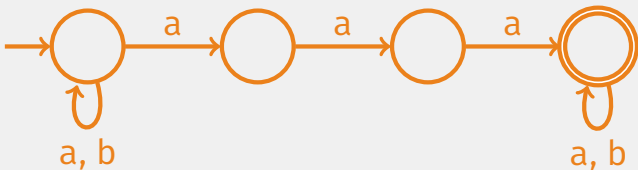
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NFA?

More examples

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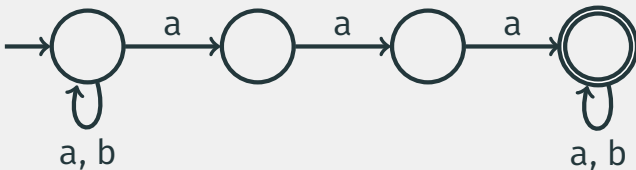
NFA?



More examples

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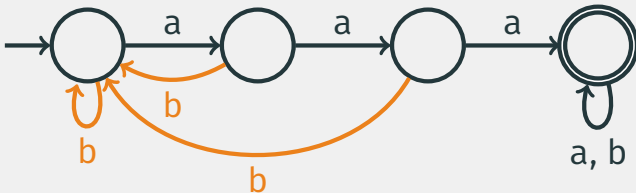
DFA?



More examples

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DFA?



More examples

$$L = \{w \in \{0, 1\}^* : w \text{ contains an even number of } 0 \text{ or} \\ \text{an odd number of } 1 \quad \}$$

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Regular expression?

More examples

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 $\text{an odd number of } 1 \quad \}$

Regular expression?

$$(1^*01^*0)^*1^* + (0^*10^*1^*)^*0^*1^*0^*$$

More examples

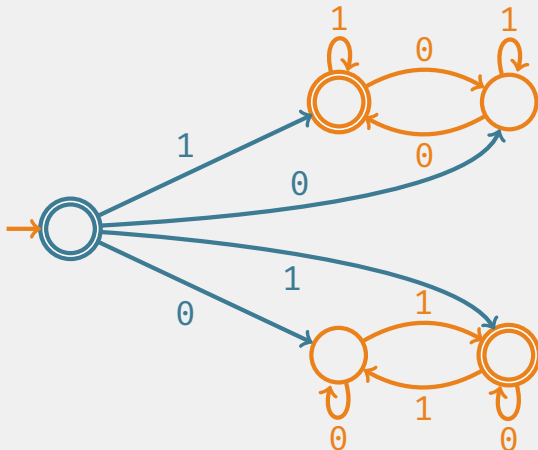
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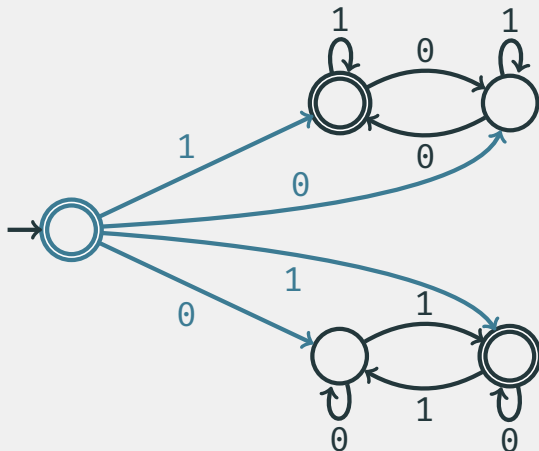
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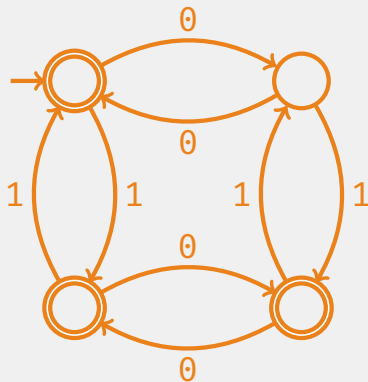
DFA?



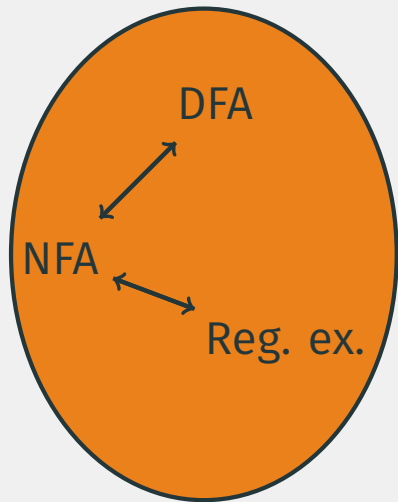
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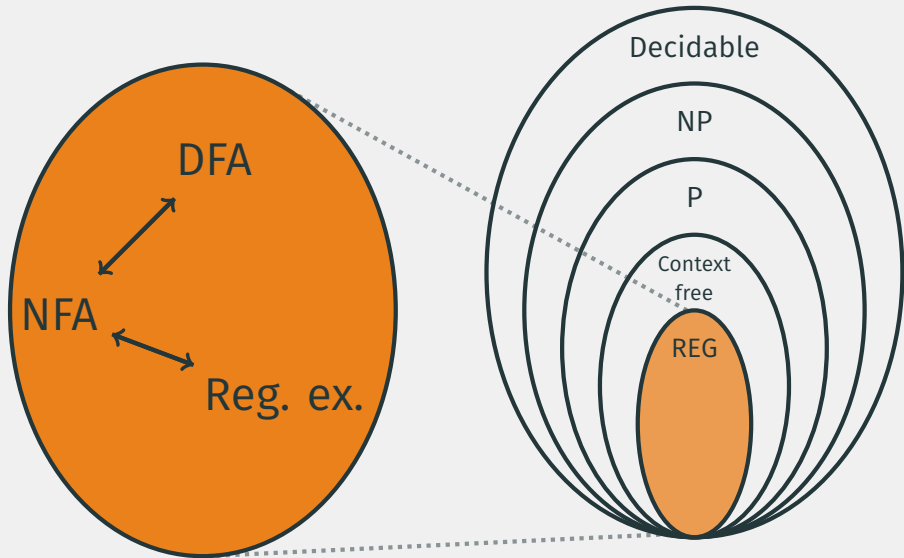
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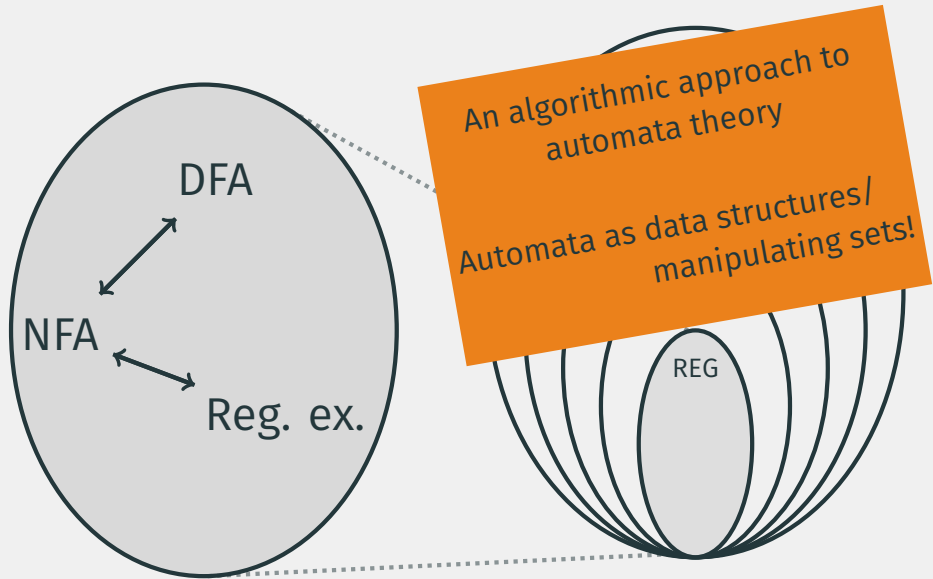
Regular languages



Regular languages



Regular languages

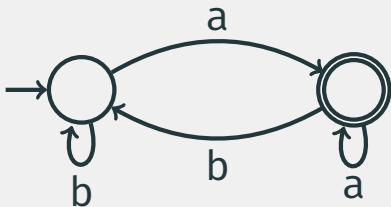


Beyond finite words

Büchi automata

An *infinite word* is an infinite sequence $a_0a_1a_2 \dots$ over some Σ

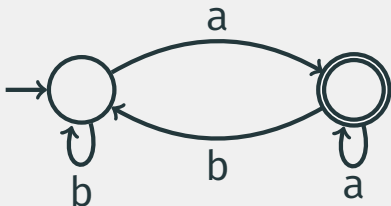
A *Büchi automaton* is "as an NFA", but accepts infinite words



Büchi automata

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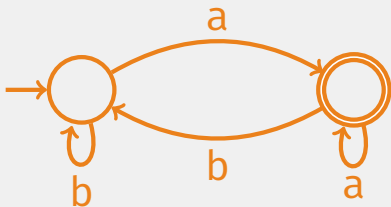
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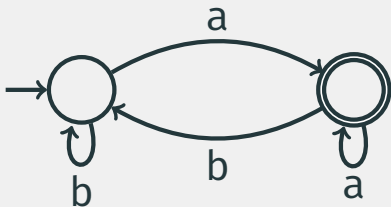
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$L_\omega(A) = \{w \in \{a, b\}^\omega : w \text{ contains infinitely many } a\}$

Büchi automata

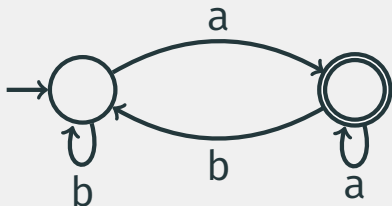
An infinite

over some Σ

A Büchi

ite words

Coming later this semester!



$$L_\omega(A) = \{w \in \{a, b\}^\omega : w \text{ contains infinitely many } a\}$$