Technische Universität München **Winter term 2017/18**
I7
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Automata and Formal Languages — Homework 14

Due 06.02.2018

Exercise 14.1
Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give LTL formulas for the following ω-languages:

(a) $\{p, q\}\, \emptyset\, \Sigma^\omega$

(b) $\Sigma^* \,\{q\}^\omega$

(c) $\Sigma^* \,(\{p\} + \{p, q\})\, \Sigma^* \,\{q\}\, \Sigma^\omega$

(d) $\{p\}^* \,\{q\}^* \,\emptyset^\omega$

Exercise 14.2
Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give Büchi automata for the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

(b) $(\mathbf{GF}p) \to (\mathbf{F}q)$

(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p\ \mathbf{U}\ (p \to q))$

(e) $\mathbf{F}q \to (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

Exercise 14.3
Let $A = (Q, \Sigma, \delta, q_0, F)$ be an automaton such that $Q = P \times [n]$ for some finite set P and $n \geq 1$. Automaton A models a system made of n processes. A state $(p, i) \in Q$ represents the current global state p of the system, and the last process i that was executed.

We define two predicates exec_j and enab_j over Q indicating whether process j is respectively executed and enabled. More formally, for every $q = (p, i) \in Q$ and $j \in [n]$, let

$$\mathrm{exec}_j(q) \iff i = j,$$
$$\mathrm{enab}_j(q) \iff (p, i) \to (p', j) \text{ for some } p' \in P.$$

(a) Give LTL formulas over Q^ω for the following statements:

  (i) All processes are executed infinitely often.

  (ii) If a process is enabled infinitely often, then it is executed infinitely often.

  (iii) If a process is eventually permanently enabled, then it is executed infinitely often.

(b) The three above properties are known respectively as *unconditional*, *strong* and *weak* fairness. Show the following implications, and show that the reverse implications do not hold:

$$\text{unconditional fairness} \implies \text{strong fairness} \implies \text{weak fairness.}$$

Exercise 14.4

Let AP $= \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. An LTL formula is a tautology if it is satisfied by all computations. Which of the following LTL formulas are tautologies?

(a) $\mathbf{G}p \rightarrow \mathbf{F}p$

(b) $\mathbf{G}(p \rightarrow q) \rightarrow (\mathbf{G}p \rightarrow \mathbf{G}q)$

(c) $\mathbf{FG}p \vee \mathbf{FG}\neg p$

(d) $\neg\mathbf{F}p \rightarrow \mathbf{F}\neg\mathbf{F}p$

(e) $(\mathbf{G}p \rightarrow \mathbf{F}q) \leftrightarrow (p\ \mathbf{U}\ (\neg p \vee q))$

(f) $\neg(p\ \mathbf{U}\ q) \leftrightarrow (\neg p\ \mathbf{U}\ \neg q)$

(g) $\mathbf{G}(p \rightarrow \mathbf{X}p) \rightarrow (p \rightarrow \mathbf{G}p)$