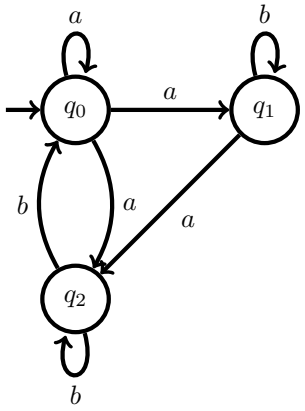
Technische Universität München **Winter term 2017/18**
I7
Prof. J. Esparza / Dr. M. Blondin **12.01.2018**

Automata and Formal Languages — Homework 12

Due 23.01.2018

Exercise 12.1
Consider the following automaton A:



(a) Interpret A as a Muller automaton with acceptance condition $\{\{q_1\}, \{q_0, q_2\}\}$. Use algorithms *NMAtoNGA* and *NGAtoNBA* from the lecture notes to construct a Büchi automaton that recognizes the same language as A.

(b) Interpret A as a Rabin automaton with acceptance condition $\{\langle\{q_0, q_2\}, \{q_1\}\rangle\}$. Follow the approach presented in class to construct a Büchi automaton that recognizes the same language as A.

Exercise 12.2

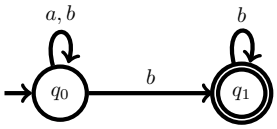
(a) Give deterministic Büchi automata for L_a, L_b, L_c where $L_\sigma = \{w \in \{a, b, c\}^\omega : w \text{ contains infinitely many } \sigma\text{'s}\}$, and intersect these automata.

(b) Give Büchi automata for the following ω-languages:

- $L_1 = \{w \in \{a, b\}^\omega : w \text{ contains infinitely many } a\text{'s}\}$,
- $L_2 = \{w \in \{a, b\}^\omega : w \text{ contains finitely many } b\text{'s}\}$,
- $L_3 = \{w \in \{a, b\}^\omega : \text{each occurrence of } a \text{ in } w \text{ is followed by a } b\}$,

and intersect these automata.

Exercise 12.3
Consider the following Büchi automaton over $\Sigma = \{a, b\}$:



(a) Sketch dag($abab^\omega$) and dag($(ab)^\omega$).

(b) Let r_w be the ranking of dag(w) defined by

$$r_w(q,i) = \begin{cases} 1 & \text{if } q = q_0 \text{ and } \langle q_0, i\rangle \text{ appears in dag}(w), \\ 0 & \text{if } q = q_1 \text{ and } \langle q_1, i\rangle \text{ appears in dag}(w), \\ \bot & \text{otherwise.} \end{cases}$$

Are r_{abab^ω} and $r_{(ab)^\omega}$ odd rankings?

(c) Show that r_w is an odd ranking if and only if $w \notin L_\omega(B)$.

(d) Construct a Büchi automaton accepting $\overline{L_\omega(B)}$ using the construction seen in class. [Hint:]